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LABORATORY
INDEXED

Wad 662/26

COLD WORKING OF CANNON
(Autofrettage)

DESIGN DATA

15-61-GX-OC

PART I

The following data for Gun Tubes and High Pressure Vessels are forwarded to the historical section so that they may serve as text material to assist in teaching future ordnance officers.

The calculations are part of ordnance achievement for World War II.

Approved for release by the
National Security Council
on 10-10-1964

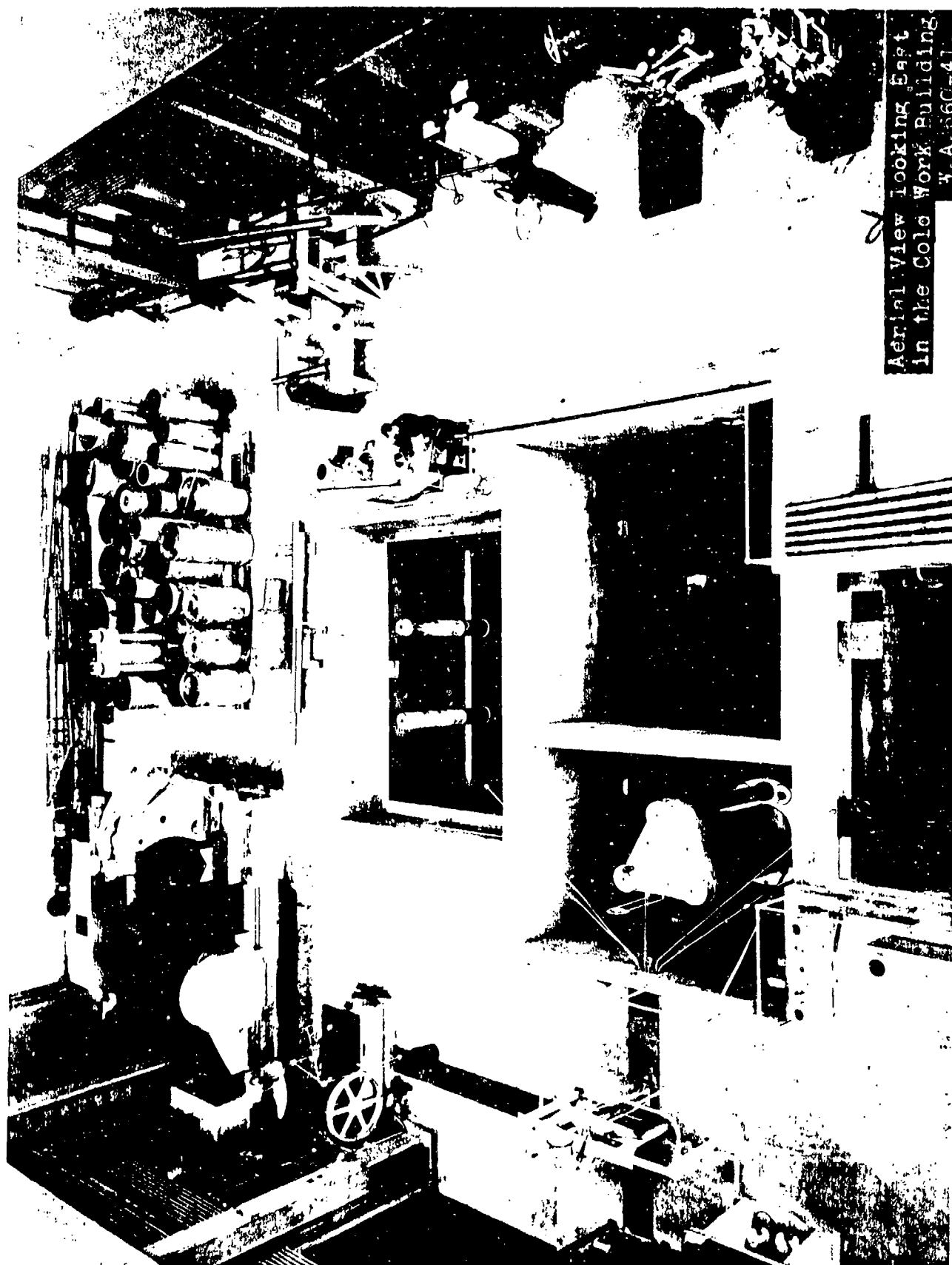
December 1943

WATERTOWN ARSENAL
WATERTOWN, MASS.

WATERTOWN ARSENAL

COLD WORKING OF CANNON
(Autofrettage)

December 1943



Aerial View Looking East
in the Cold War Building
W.A. 860-41

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Cold Working of Cannon
(Autofrettage)

The first recorded enlargement of the bore of gun barrels by interior pressure occurred about one hundred years ago at Springfield Armory. Each rifle barrel was tested by firing a special cartridge loaded to produce a pressure one and one-half times the service pressure. The cartridge was fired before the barrel was bored and chambered, and the only purpose was to insure rupture of any defective barrels. Barrels which withstood this treatment were accepted for use. It was observed that both the interior and exterior diameters were enlarged by the high pressure, but it was not detected that this cold stretching of the metal increased the elastic strength. Not until the Krag-Jorgensen magazine rifle was adopted, was it established that this enlargement of the bore was accompanied by a marked increase in the elastic strength of the gun.

After World War I, a study of the possible application of the process to the manufacture of cannon was made at Watertown Arsenal. The study showed that successful application was dependent on the practicability of making equipment that could generate, transmit into and retain in the bore, the high pressure required. Major General C. C. Williams, then Chief of Ordnance, authorized the making of the necessary equipment and when the tests made showed promise, ordered Watertown Arsenal to conduct such experiments as would determine if cold worked single piece

forgings could be made good enough to replace the "built-up by shrinkage" and the "wire wrapped" cannon heretofore used.

The experiments (in the early 1920's) directed by Brigadier General T. C. Dickson, then Commanding Watertown Arsenal, were conducted by Dr. F. C. Langenberg and Mr. J. C. Solberg (now Lt. Col. Solberg) and later by Captain S. B. Ritchie (now Colonel Ritchie) and Mr. H. C. Mann.

"Cold Working" or permanently enlarging all diameters of a cylinder by interior pressure produces an internal strain of tension in the exterior portion of the wall. In a built-up gun, the tube is put into a state of compression by the jackets and hoops. "Cold Working" produces this effect without hoops and the method as originally developed was termed "Autofrettage" which, literally translated from the French means "self-hooping."

The adoption of the cold working process to guns makes it possible to make them in a single piece and saves the accurate machining previously required on the exterior surface for shrinkage operation. This process subjects every fiber of the metal to a pressure far in excess of any which the metal will receive in service. As a "proof-test" this feature is of great value because the strength of the gun has actually been "proved" prior to any firing.

The process was first applied to forged gun tubes and later to centrifugally cast gun tubes. Much of the credit for the application of the process to mass production belongs to Captain D. H. Newhall, Ordnance Department, who has been in charge of the Cold-

Working Department at Watertown Arsenal immediately preceding and during World War II.

The equipment at Watertown Arsenal has been designed to develop hydraulic pressures up to 150,000 pounds per square inch. The hydraulic medium is water containing soluble oil and tri-sodium phosphate (as a rust inhibitor). The pressure is built up in two stages. An ordinary commercial pump capable of building up a pressure of 10,000 pounds per square inch comprised the first stage. The second stage employs a unit known as an intensifier which builds up the pressure required. The comparatively low pressure from the commercial pump is introduced into a large area on a floating piston in the intensifier. High pressure is taken off the other side of the floating piston which has a much smaller area. The high pressure developed is in inverse proportion to the areas.

The formulae and dissertation appearing on the following pages covering Design data for gun tubes and high pressure vessels, are by Captain D. H. Newhall, Ordnance Department, Officer in Charge of the Cold-Work Section at Watertown Arsenal. Not only do they constitute a distinguished contribution to the War effort, but will serve as valuable text material for future Ordnance Officers. All the calculations involved have been checked by personnel from the Research Division, Watertown Arsenal Laboratory.

FOREWORD

In the design of cold-worked gun tubes and high-pressure vessels, the designer is confronted with a relatively large number of fairly complex formulae concerning the mechanics of thick hollow cylinders. To simplify their use, the Cold Work Section at Watertown Arsenal has found it convenient to tabulate these formulae on simple charts, and in addition, to present certain of them in curve form.

In the course of discussion with other Ordnance Department design agencies, several requests for the tabulations and curves were received and to meet these requests the material has been collected. After collecting the formulae and curves, which, by themselves demanded some explanation, it occurred to the author that the presentation could be made more interesting and understandable if it were expanded somewhat to include a discussion of the general behavior of cylinders subjected to pressure. This report was ~~therefore~~ expanded accordingly and organized to correspond to the four (4) distinct conditions or states of stress that could be encountered, namely:

1. Elastic State;
2. Theories Concerning Initial Yielding;
3. Plastic State;
4. Rupture State;

Even though expanded the main purpose of this report is to present useable formulae and curves in concise form and for this reason certain fundamental derivations have been omitted. Space is not given to crediting the source of much of the material. Some formulae, altered only slightly from their original form, should be credited to standard texts on strength of materials. Explanatory notes have been kept to a minimum. Some points discussed in the

section concerning the Plastic State are subject to controversy and claim is not made as to their final and absolute accuracy. All data presented, however, have actually been applied to design and related engineering problems in connection with cold-working and have proven to be sufficiently accurate for all practical purposes. Still other points merely report observations made by the author during his contact with high pressure work at Watertown Arsenal.

It is frequently necessary to calculate elastic stresses and strains resulting from odd and complex systems of loading. While it would be possible to derive formulae for the many cases that could exist, the author has found it more convenient to plot or express only the simpler relationships, resorting to the principle of superposition for the more complex ones. In using this principle a complex loading is broken down into separate simpler sets of loads. The effect of each load is determined, with an algebraic summation being then made to give the net effect.

In the Ordnance Department, the original work on material in the Plastic State was done at Watertown Arsenal in the early 1920's by Dr. F. C. Langenberg and Mr. J. C. Solberg (now Lt. Colonel Solberg, Ordnance Department) under the direction of the late General Tracy Dickson. The "Flow Factor" and "Pressure Factor" relationships, discussed in the text, were among their many contributions.

Their work has been extended in that equations have now been derived covering strain distribution in both transverse and longitudinal planes. This in turn led to the development of an equation for longitudinal shrinkage in a tapered tube. These equations, without their derivations have been included.

DESIGN DATA
for
GUN TUBES AND HIGH PRESSURE VESSELS

By
Donald H. Newhall
Capt. Ord. Dept.

N O M E N C L A T U R E

- S_{yp} = Stress at yield, pounds per square inch
 S_t = Stress in the tangential direction, pounds per square inch
 S_r = Stress in the radial direction, pounds per square inch
 S_z = Stress in the longitudinal direction, pounds per square inch
 e_t = Strain in the tangential direction, inches per inch
 e_r = Strain in the radial direction, inches per inch
 e_z = Strain in the longitudinal direction, inches per inch
 e_i = Strain on the ID, inches per inch
 e_o = Strain on the OD, inches per inch
 E = Young's Modulus of Elasticity, pounds per square inch
 μ = Poisson's Ratio
 P = Pressure, pounds per square inch
 P_i = Pressure on the bore, pounds per square inch
 P_o = Pressure on the outside surface, pounds per square inch
 $W = \text{Wall ratio} = \frac{\text{Outside Diameter}}{\text{Inside Diameter}}$
 W_m = Wall ratio at the muzzle
 W_b = Wall ratio at the breech
 $K = \frac{W^2 + 1}{W^2 - 1}$
 r = General radius in a thick hollow cylinder
 a = Bore radius in a thick hollow cylinder
 b = Outside radius of a thick hollow cylinder
 $FF = \text{Flow factor} = \frac{\Delta ID}{\Delta OD}$
 $PF = \text{Pressure factor} = \frac{\text{Pressure}}{\text{Stress at yield}}$
 \ln = Natural Logarithm

ELASTIC STATE - WITHOUT LONGITUDINAL LOADING

Fig. 1 (Equations 1 - 42 inclusive) lists a collection of pertinent formulae covering stress and strain in thick hollow cylinders assuming that there is no longitudinal load applied and that cylinders are free to extend or contract.

Equations 1, 2 and 3 (Fig. 1) represent the well known fundamental equations of stress as derived originally by Lamé and rewritten for the sake of convenience in terms of wall ratio.

Equations 4, 5 and 6 (Fig. 1) are fundamental expressions of Hooke's Law of Deformation for material subjected to combined stress.

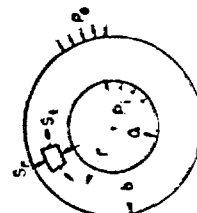
The fundamental equations (1-6) apply when internal and external pressures exist simultaneously. For the case when internal pressure only is used, these equations may be reduced to the general ones cited in equations 7 - 12 inclusive. Likewise in the case of external pressure only, the general equations 25 - 30 apply.

In making calculations, a designer is usually most interested in the stresses and strains at the boundaries of the cylinder (at bore and outside surfaces) and for these specific cases the formulae above further reduce to those shown as equations 13 - 24 (internal pressure only) and equations 31 - 42 (external pressure only). These specific equations are plotted against wall ratio (Figs. 2 - 8 inclusive) with the plot limited to those values of wall ratio normally encountered in gun tubes and high-pressure vessel design. Those equations concerning stress were solved and plotted assuming an applied pressure of 10,000 psi. Since stress and pressure in the elastic state are directly proportional, stresses prevailing at other pressures are readily computed. In a similar manner, equations concerning strain were plotted assuming a pressure of 100,000 psi.

In the equations discussed above, the longitudinal stress (S_z) has been assumed to be zero. This has been a controversial point in the field of mechanics and it is interesting to note that Capt. Elmo Mathews has apparently confirmed the assumption in his M. I. T. thesis "Photoelastic Examination of Stress in Thick Hollow Cylinders". (W. A. #660/12)

COLLECTED STRESS AND STRAIN FORMULAE FOR THICK HOLLOW CYLINDERS IN THE ELASTIC STATE

	TANGENTIAL STRESS S_t	RADIAL STRESS S_r	LONGITUDINAL STRESS S_z	TANGENTIAL STRAIN ϵ_t	RADIAL STRAIN ϵ_r	LONGITUDINAL STRAIN ϵ_z
INTERNAL & EXTERNAL PRESSURE						
1. FUNDAMENTAL	$S_t = \frac{P_i \left(1 + \frac{b^2}{r^2}\right) - W^2 P_o \left(1 + \frac{a^2}{r^2}\right)}{W^2 - 1}$	$S_r = \frac{P_i \left(1 - \frac{b^2}{r^2}\right) - W^2 P_o \left(1 - \frac{a^2}{r^2}\right)}{W^2 - 1}$	$S_z = 0$	$\epsilon \epsilon_t = S_t - \mu (S_r + S_z)$	$\epsilon \epsilon_r = S_r - \mu (S_t + S_z)$	$\epsilon \epsilon_z = S_z - \mu (S_t + S_r)$
2. GENERAL	$S_t = \frac{P_i}{W^2 - 1} \left(1 + \frac{b^2}{r^2}\right)$	$S_r = \frac{P_i}{W^2 - 1} \left(1 - \frac{b^2}{r^2}\right)$	$S_z = 0$	$\epsilon \epsilon_t = \frac{P_i}{W^2 - 1} \left[(1 - \mu) + \frac{b^2}{r^2} (1 + \mu) \right]$	$\epsilon \epsilon_r = \frac{P_i}{W^2 - 1} \left[(1 - \mu) - \frac{b^2}{r^2} (1 + \mu) \right]$	$\epsilon \epsilon_z = \frac{2 \mu P_i}{W^2 - 1}$
3. AT P.O.	$S_t = P_i \frac{W^2 + 1}{W^2 - 1}$	$S_r = -P_i$	$S_z = 0$	$\epsilon \epsilon_t = \frac{P_i}{W^2 - 1} \left(7 + 1.3 \frac{b^2}{r^2} \right)$	$\epsilon \epsilon_r = \frac{P_i}{W^2 - 1} \left(7 - 1.3 \frac{b^2}{r^2} \right)$	$\epsilon \epsilon_z = \frac{-6 P_i}{W^2 - 1}$
4. AT P.b	$S_t = \frac{2 P_i}{W^2 - 1}$	$S_r = 0$	$S_z = 0$	$\epsilon \epsilon_t = \frac{2 P_i}{W^2 - 1}$	$\epsilon \epsilon_r = \frac{-6 P_i}{W^2 - 1}$	$\epsilon \epsilon_z = \frac{-6 P_i}{W^2 - 1}$
EXTERNAL PRESSURE ONLY						
5. GENERAL	$S_t = \frac{-P_o W^2}{W^2 - 1} \left(1 + \frac{a^2}{r^2}\right)$	$S_r = \frac{-P_o W^2}{W^2 - 1} \left(1 - \frac{a^2}{r^2}\right)$	$S_z = 0$	$\epsilon \epsilon_t = \frac{-P_o W^2}{W^2 - 1} \left[(1 - \mu) + \frac{a^2}{r^2} (1 + \mu) \right]$	$\epsilon \epsilon_r = \frac{-P_o W^2}{W^2 - 1} \left[(1 - \mu) - \frac{a^2}{r^2} (1 + \mu) \right]$	$\epsilon \epsilon_z = \frac{-2 \mu P_o W^2}{W^2 - 1}$
6. AT P.O.	$S_t = \frac{-2 P_o W^2}{W^2 - 1}$	$S_r = 0$	$S_z = 0$	$\epsilon \epsilon_t = \frac{-2 P_o W^2}{W^2 - 1} \left(7 + 1.3 \frac{a^2}{r^2} \right)$	$\epsilon \epsilon_r = \frac{-2 P_o W^2}{W^2 - 1} \left(7 - 1.3 \frac{a^2}{r^2} \right)$	$\epsilon \epsilon_z = \frac{6 P_o W^2}{W^2 - 1}$
7. AT P.b	$S_t = -P_o \left(\frac{W^2 + 1}{W^2 - 1} \right)$	$S_r = -P_o$	$S_z = 0$	$\epsilon \epsilon_t = \frac{-P_o}{W^2 - 1} (7 W^2 + 1.3)$	$\epsilon \epsilon_r = \frac{-P_o}{W^2 - 1} (7 W^2 - 1.3)$	$\epsilon \epsilon_z = \frac{-6 P_o W^2}{W^2 - 1}$



$W = \frac{b}{a}$ - WALL RATIO

S_t - STRESS IN THE TANGENTIAL DIRECTION
 S_r - STRESS IN THE RADIAL DIRECTION
 S_z - STRESS IN THE LONGITUDINAL DIRECTION
 ϵ_t - STRAIN IN THE TANGENTIAL DIRECTION
 ϵ_r - STRAIN IN THE RADIAL DIRECTION

ϵ_z - STRAIN IN THE LONGITUDINAL DIRECTION

P_i - PRESSURE IN THE BORE, P.S.I.

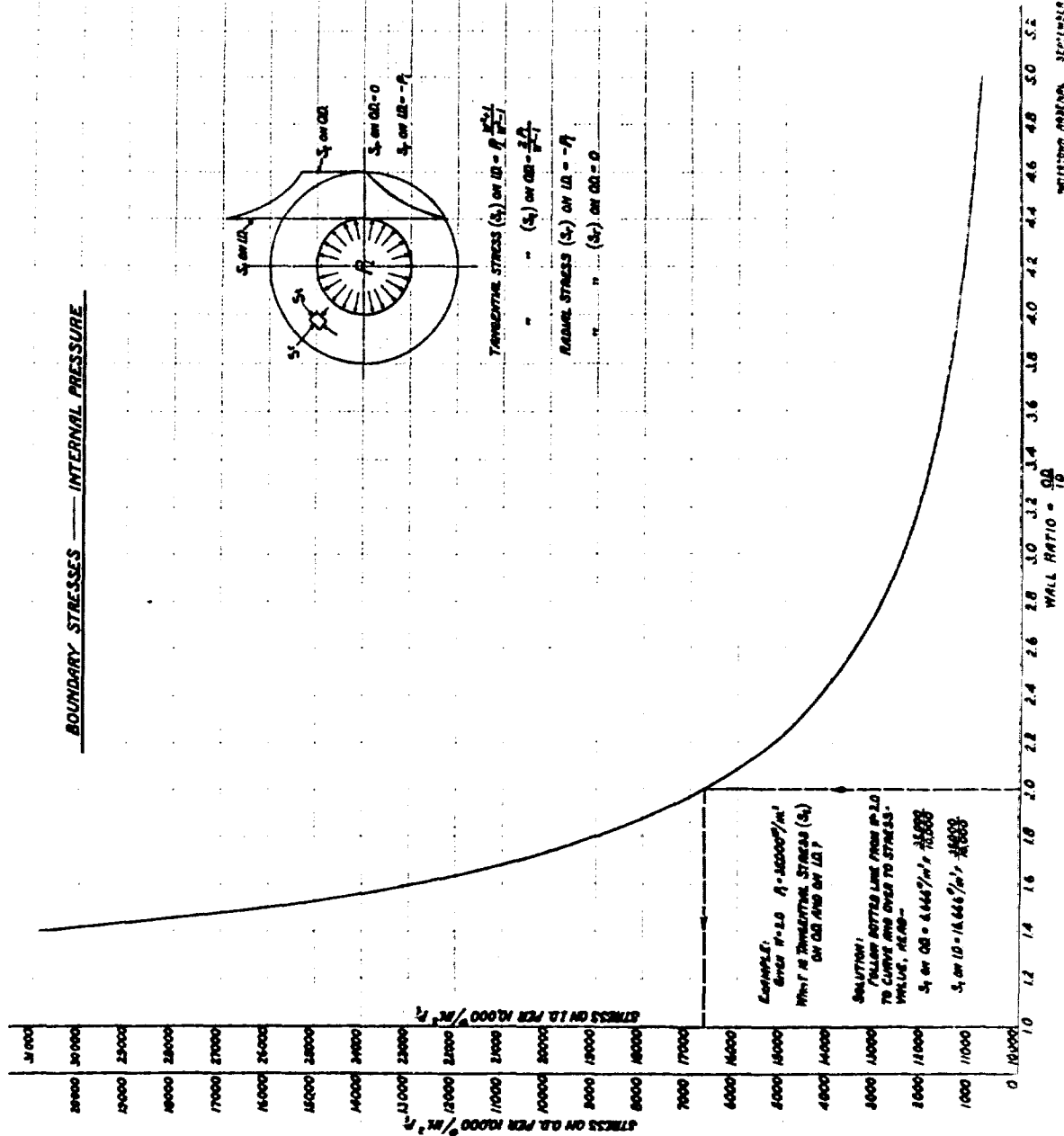
P_o - PRESSURE ON THE OUTSIDE, P.S.I.

E - MODULUS OF ELASTICITY - 30,000,000 P.S.I. FOR STEEL

μ - POISSON'S RATIO - 0.3 FOR STEEL

WATERTOWN ARSENAL SEPT. 11, 1942
 DRW. BY: A.J.B. APP'D BY: J.A.C.W.
 CAPT ORD DEPT, U.S.A.

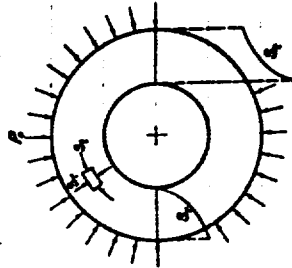
FIGURE 1



WASHINGTON AREA, SEPTEMBER 15, 1962
DOW BY A.P.P. APP. 81

BOUNDARY STRESSES — EXTERNAL PRESSURE

BOTH S_1 AND S_2 ARE COMPRESSIVE STRESSSES.



TEMPERATURE	STRESS (N)	ON	OFF	STRESS (N)
0	0	0	0	0
10	10	10	10	10
20	20	20	20	20
30	30	30	30	30
40	40	40	40	40
50	50	50	50	50
60	60	60	60	60
70	70	70	70	70
80	80	80	80	80
90	90	90	90	90
100	100	100	100	100

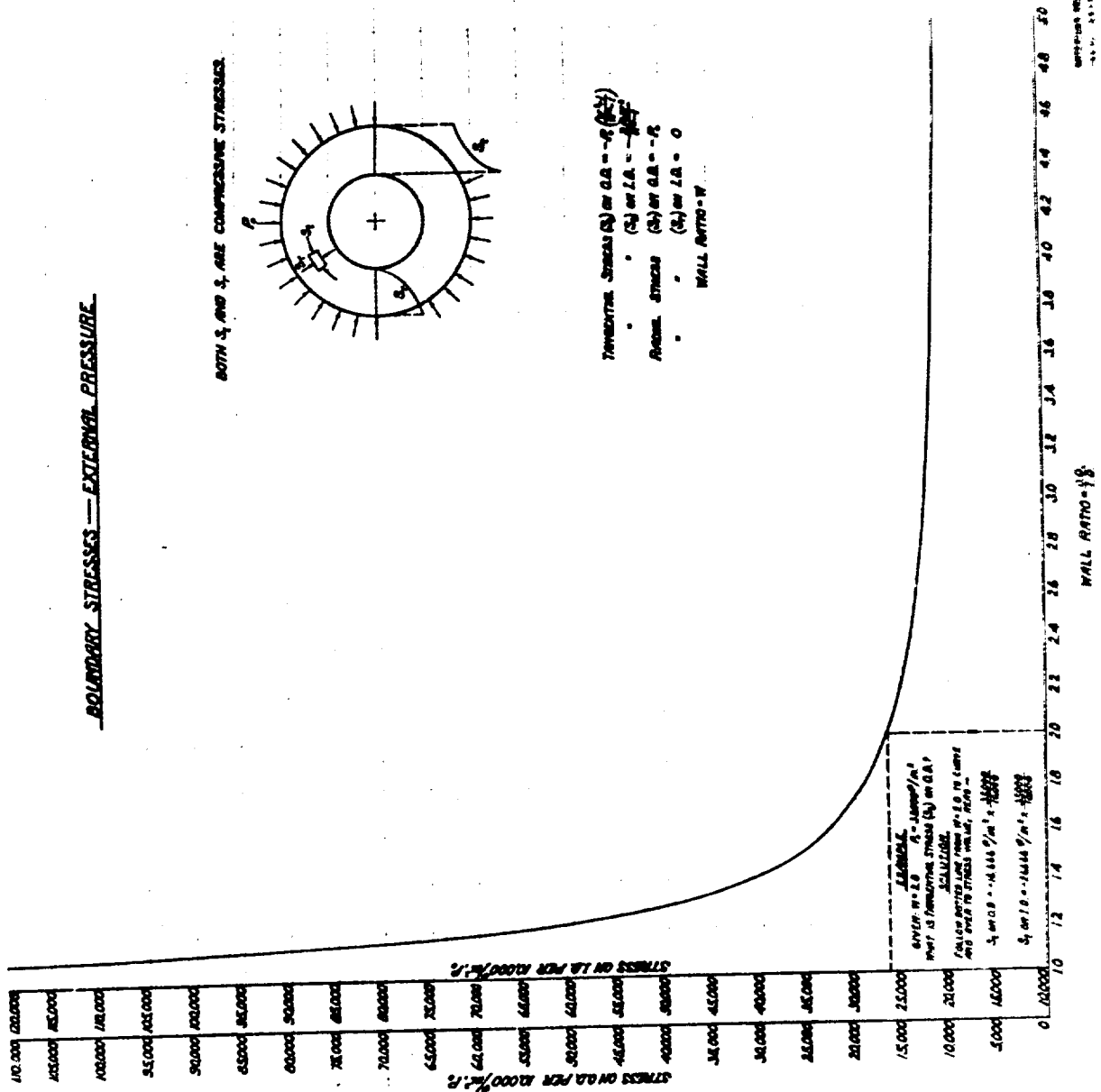


FIGURE 3

W.A.639-4259

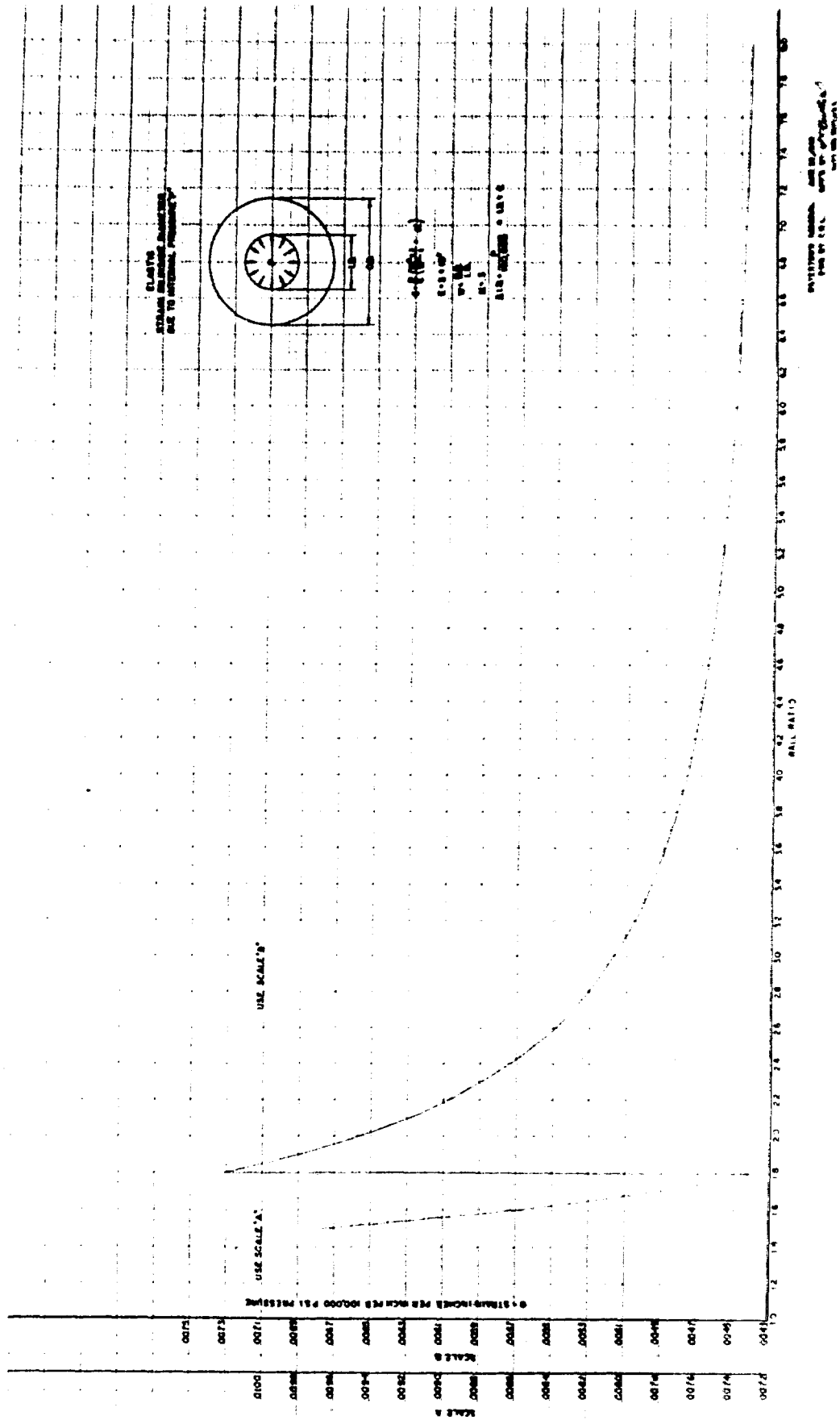
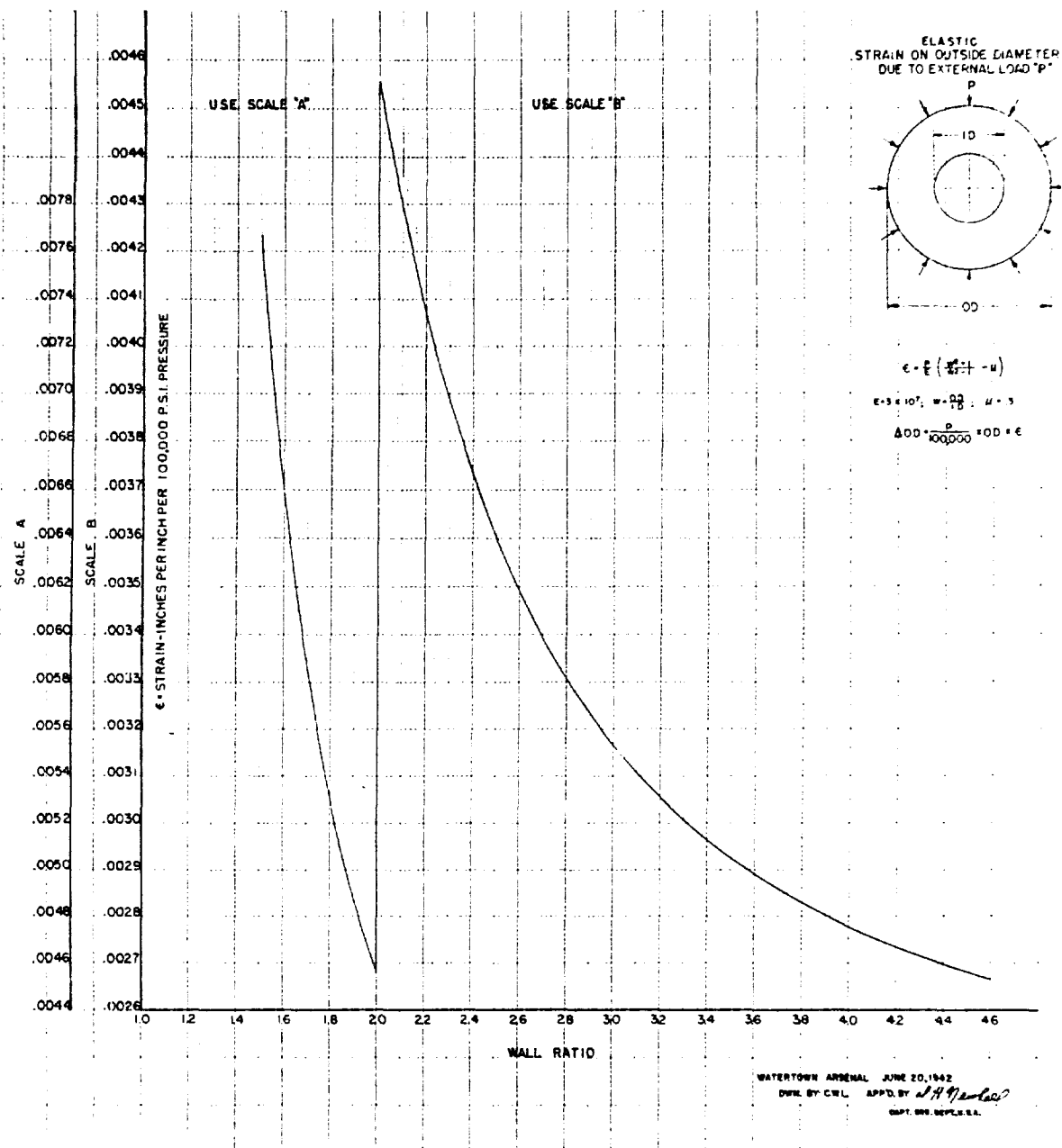


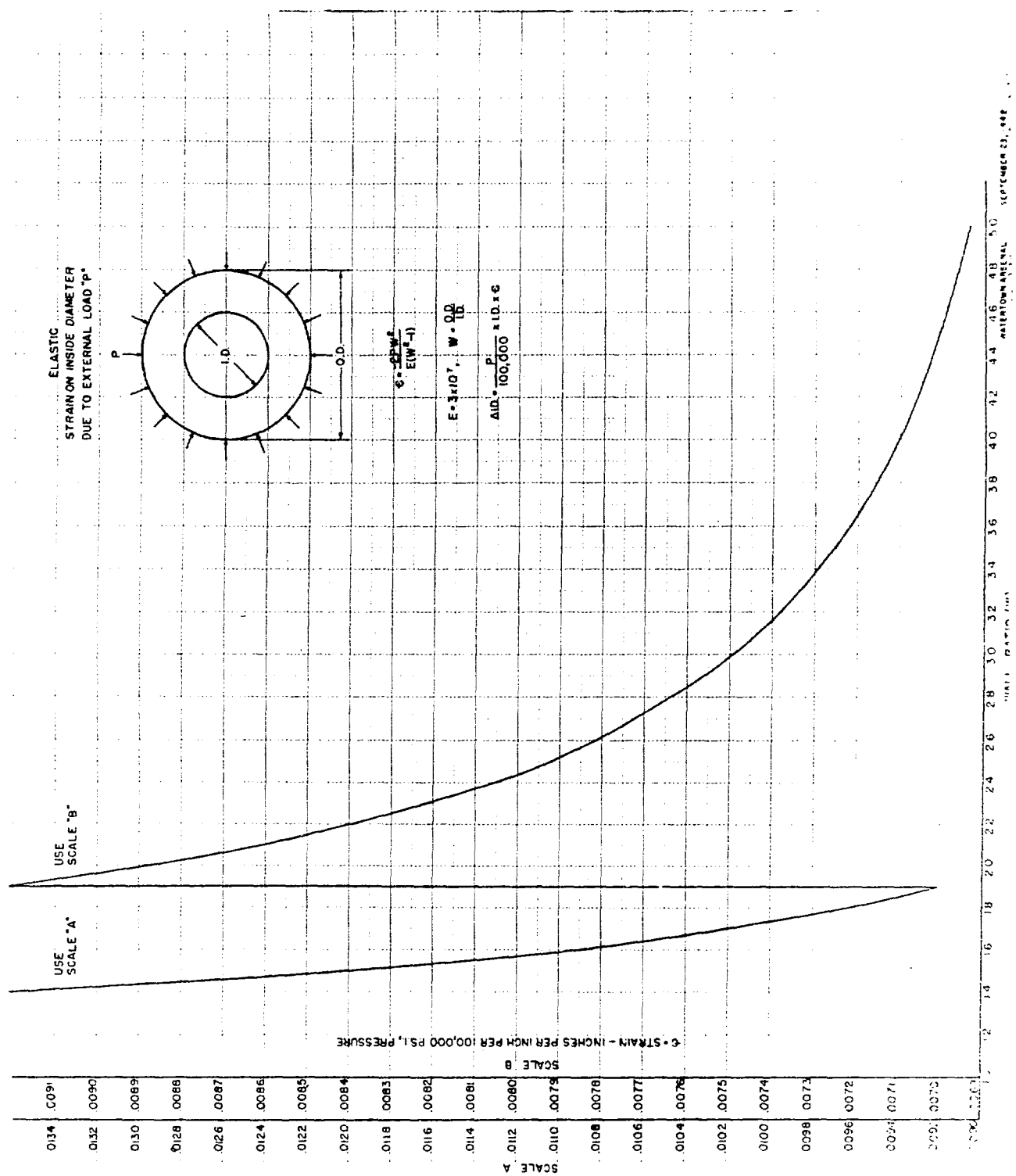
FIGURE 1



W.A. 639-4258

FIGURE 6

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FIGURE

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STRAIN IN LONGITUDINAL DIRECTION vs. WALL RATIO

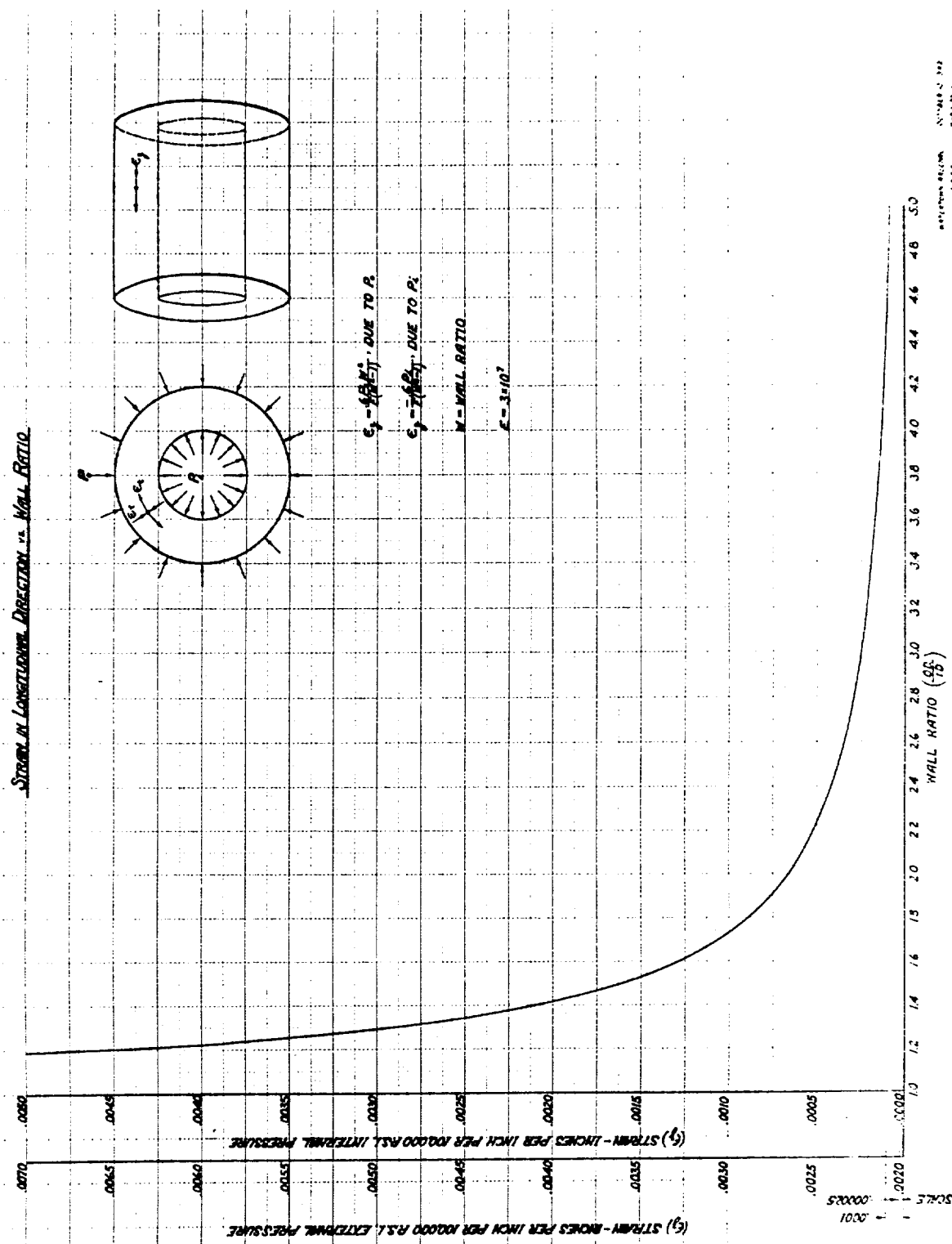


FIGURE 8

STRAIN IN LONGITUDINAL DIRECTION vs. WALL RATIO
 NUMBER 2-302
 MAY 1958

ELASTIC STATE - WITH LONGITUDINAL LOADING

Special Case - Capped Cylinder

In the preceding section equations were discussed assuming that there was no longitudinal stress. This ceases to be the case when a cylinder is restrained longitudinally, for instance when the cylinder is capped. In this case the end thrust of the internal pressure is exerted as a longitudinal load and longitudinal stress becomes:

$$S_z = \frac{P}{W^2 - 1} \text{ --- (43)}$$

Longitudinal loading does not change the relationship between pressure and the transverse stresses (tangential and radial) hence the stress equations (S_t and S_r) given in Fig. 1 hold for this case as well as for the case without longitudinal loading. As indicated in Equation 43, longitudinal stress is dependent only on pressure and wall ratio and therefore is uniform across the transverse section. Fig. 9 is a plot (wall ratio vs. longitudinal stress) of Equation 43 using a unit pressure of 10,000 psi.

Equations for the principal strains (e_t , e_r and e_z) for the longitudinally loaded case are derived by substituting the above expression for longitudinal stress (Equation 43), together with the corresponding values of S_t and S_r , in the fundamental equations 4, 5 and 6 (Fig. 1). The resulting equations are tabulated below:

GENERAL

$$E e_t = \frac{P_1}{W^2 - 1} \left(.4 + 1.3 \frac{b^2}{r^2} \right) \text{ --- (44)}$$

$$E e_r = \frac{P_1}{W^2 - 1} \left(.4 - 1.3 \frac{b^2}{r^2} \right) \text{ --- (45)}$$

$$E e_z = \frac{.4P}{W^2 - 1} \text{ --- (46)}$$

WHEN $r = a$

$$E e_t = \frac{P}{W^2 - 1} \left(.4 + 1.3W^2 \right) \text{ --- (47)}$$

$$E e_r = \frac{P}{W^2 - 1} \left(.4 - 1.3W^2 \right) \text{ --- (48)}$$

$$E e_z = \frac{.4P}{W^2 - 1} \text{ --- (49)}$$

WHEN $r = b$

$$E e_t = \frac{1.7P}{W^2 - 1} \text{ --- (50)}$$

$$E e_r = \frac{-.9P}{W^2 - 1} \text{ --- (51)}$$

$$E e_z = \frac{.4P}{W^2 - 1} \text{ --- (52)}$$

It is pointed out that cases of capped cylinders could be readily calculated by the principle of superposition. The formulac are, however, included here as they are used with sufficient frequency to justify their derivation.

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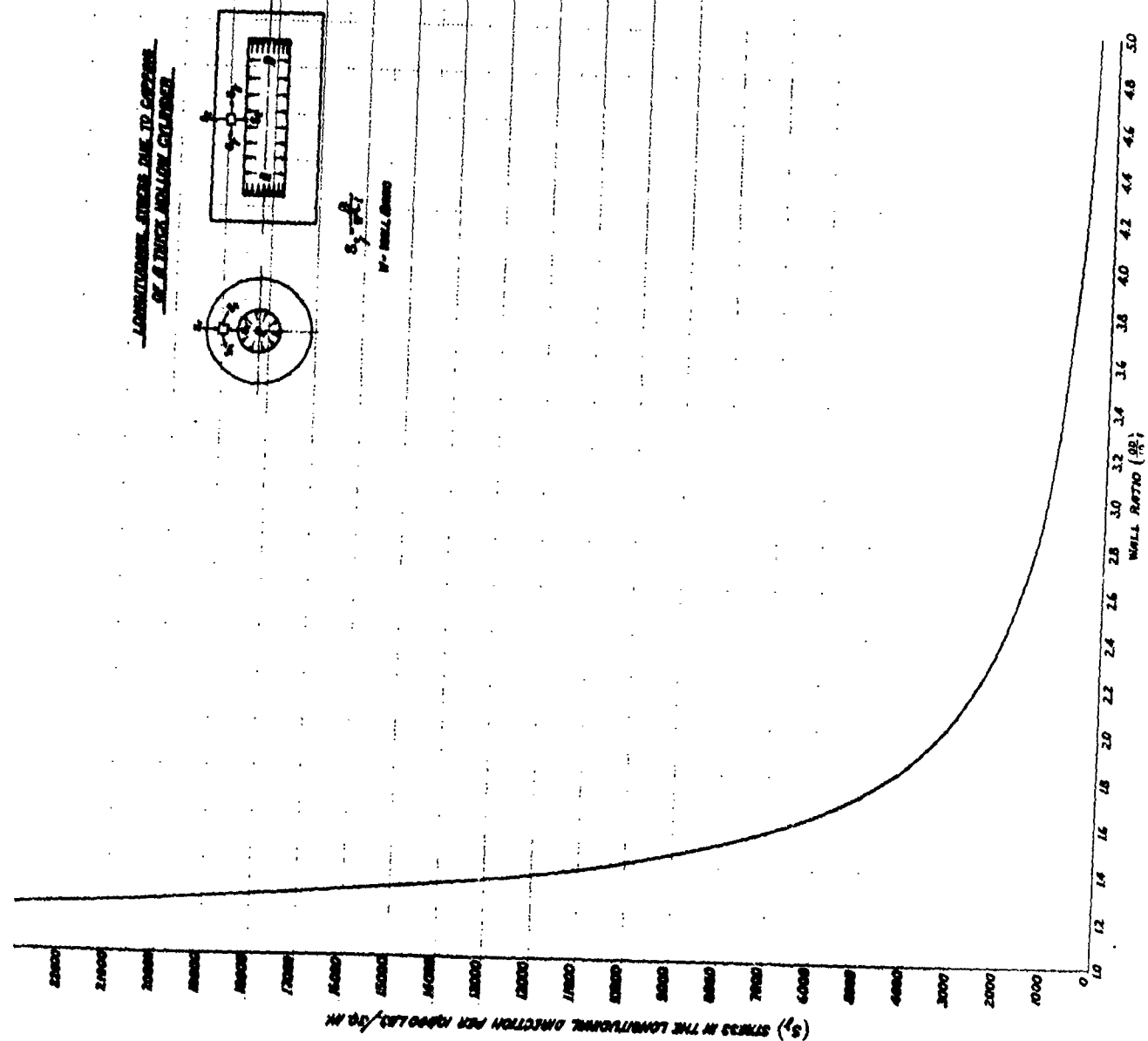


FIGURE 9

WATERMAN PUBLISHING
NEW YORK, N.Y.
1954

THEORIES CONCERNING INITIAL YIELDING

When material is loaded in one direction only, yielding occurs when the stress is equal to yield stress (uni-axial) as determined from a simple tensile test bar. This is not the case if the material is subjected to combined stress (bi-axial or tri-axial) and in accounting for such conditions of yielding various theories have been advanced. The following tabulation shows the combination of stresses that certain of the theories hold as the condition under which yielding, in cylinders subjected to internal pressure, begins: (in all cases it is assumed that there is no longitudinal stress)

Maximum Stress Theory $S_{yp} = S_t$

Maximum Strain Theory $S_{yp} = E e_t = S_t - \mu S_r$

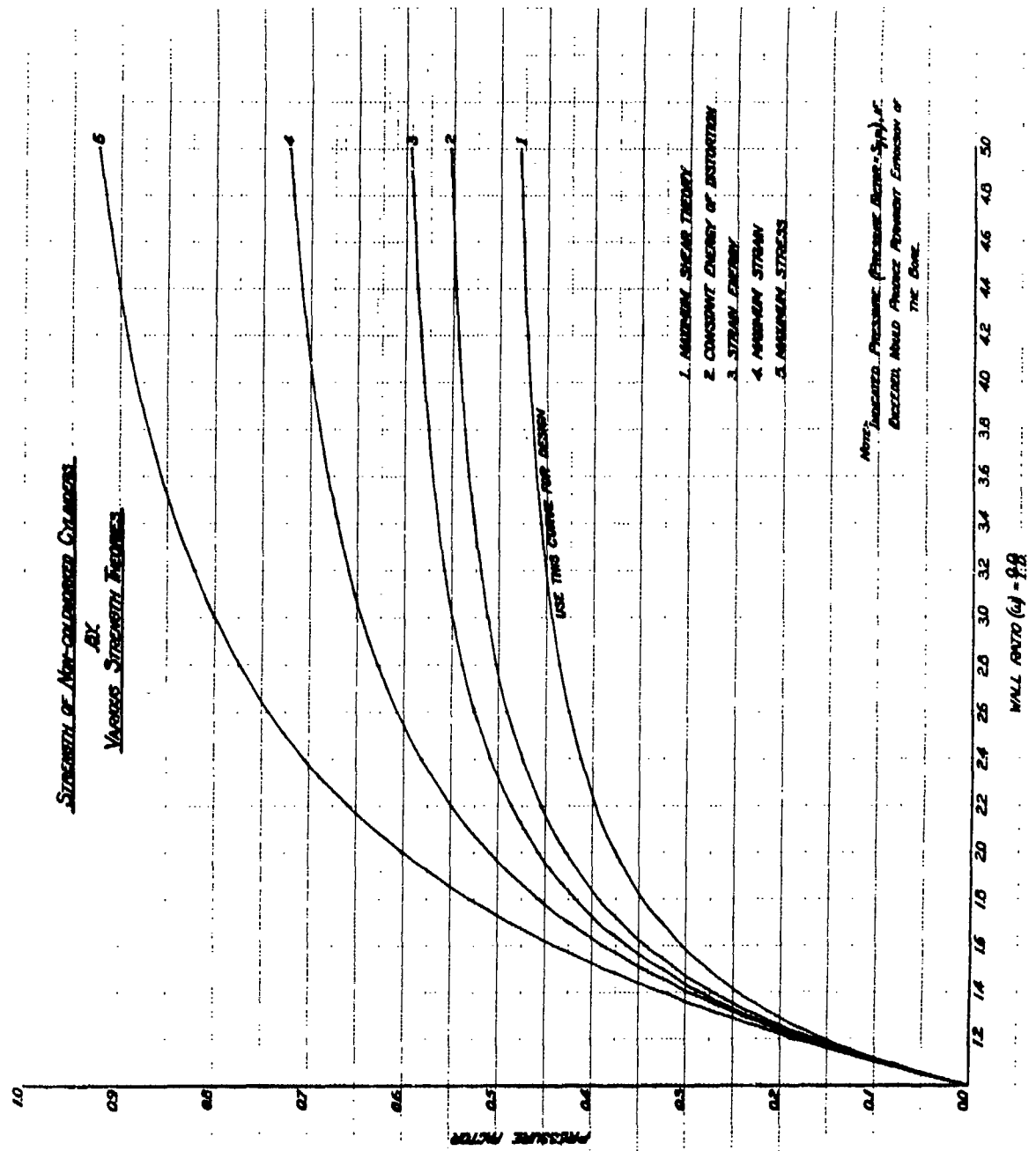
Strain Energy Theory $(S_{yp})^2 = S_t^2 + S_r^2 - 2\mu S_t S_r$

Constant Energy of Distortion
Theory (Von Mises) $2(S_{yp})^2 = (S_t - S_r)^2 + S_r^2 + S_t^2$

Maximum Shear Theory $S_{yp} = S_t - S_r$

These theories can be vividly compared when their equations are expressed in terms of pressure, yield strength and wall ratio. The equations listed below, and plotted in Fig. 10, provide this comparison. They were derived by handling the above equations in the following manner:

- 1) Letting the ratio $\frac{P}{S_{yp}} = \text{Pressure Factor}$
- 2) Substituting equations 13 and 14 (Fig. 1) as expressions for S_t and S_r
- 3) Assuming $\mu = .3$
- 4) Letting the ratio $\frac{W^2 + 1}{W^2 - 1} = K$
- 5) Solving for Pressure Factor in terms of K



VTN.639-5975

FIGURE 10

NAVY ARSENAL NOVEMBER 30, 1943
Described in "The Bone" and "The Bone"

Maximum Stress Theory	$PF = \frac{1}{K}$
Maximum Strain Theory	$PF = \frac{1}{K + .3}$
Strain Energy Theory	$PF = \frac{1}{(K^2 + .6K + 1)^{\frac{1}{2}}}$
Constant Energy of Distortion	$PF = \frac{1}{(K^2 + K + 1)^{\frac{1}{2}}}$
Maximum Shearing Stress Theory	$PF = \frac{1}{K + 1}$

Many investigators have attempted to determine which of the above theories most closely predicts the initial yielding. In ductile materials, experiments for the most part seem to agree with the constant energy of distortion theory. However, the maximum shearing stress theory does not differ greatly ($12\frac{1}{2}\%$ at $W = 2$) and since the difference is on the conservative side, it appears to be the most practical one for design work.

It is pointed out that the ratio $(-K)$, as used above, has appreciable significance in that it is the ratio of principal bore stresses (S_t / S_r). This may be shown by taking the ratio of equations 13 and 14 (Fig. 1). An interesting relationship also exists between the ratio of principal bore stresses and wall ratio. The equation and plot of this are shown in Fig. 11.

Assuming a constant internal pressure, it is seen from Fig. 11 that as values for wall ratio approach unity, the values for the ratio of bore stresses approach infinity.

Since the radial stress (S_r) at the bore remains equal to the internal pressure, the curve indicates that for small wall ratios the magnitude of the radial stress is negligible compared with the tangential stress. Tangential stress, therefore, tends to become the only principal stress that needs to be considered. The so-called "boiler" formulae used in the design of thin-walled pressure vessels, neglect radial stress and with Fig. 11 showing the influence of wall ratio, one can appreciate that the boiler formulae are limited in their application.

Again referring to Fig. 11, it is seen that as values for wall ratio increase, the values for the ratio of bore stresses approach minus one asymptotically. Minus one, as is well known, is the value for the ratio of principal stresses in material subjected to pure shear. Experimental work shows that the cylinders with large wall ratios yield approximately in agreement with the maximum shearing stress theory and, in view of the above, this is not surprising.

It is to be noted that in the above comparison of theories, a variable termed "pressure factor" was used. This, as will be shown in the next section, is a convenient dimensionless ratio used extensively in comparing the pressure characteristics of thick hollow cylinders.

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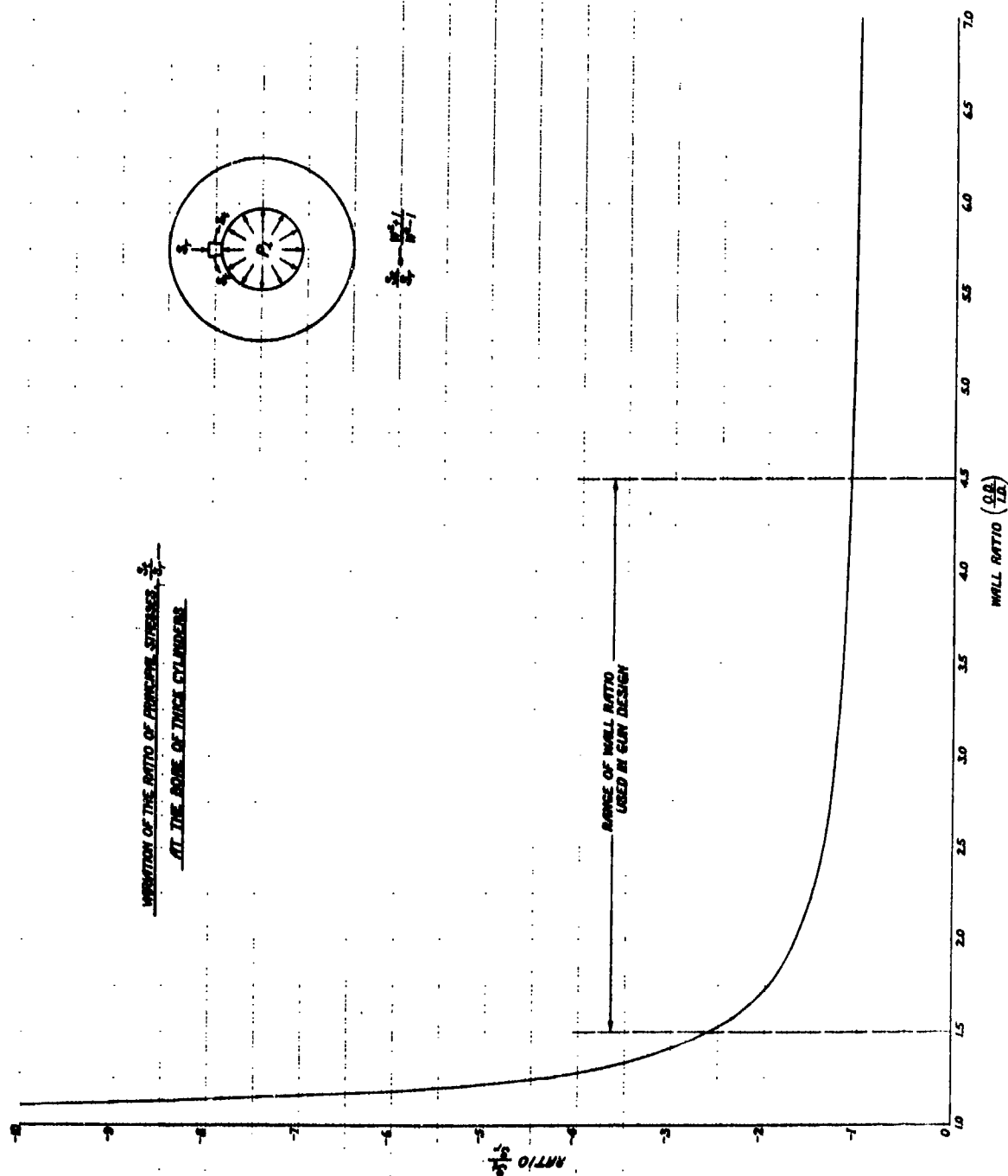


FIGURE 11

PLASTIC STATE

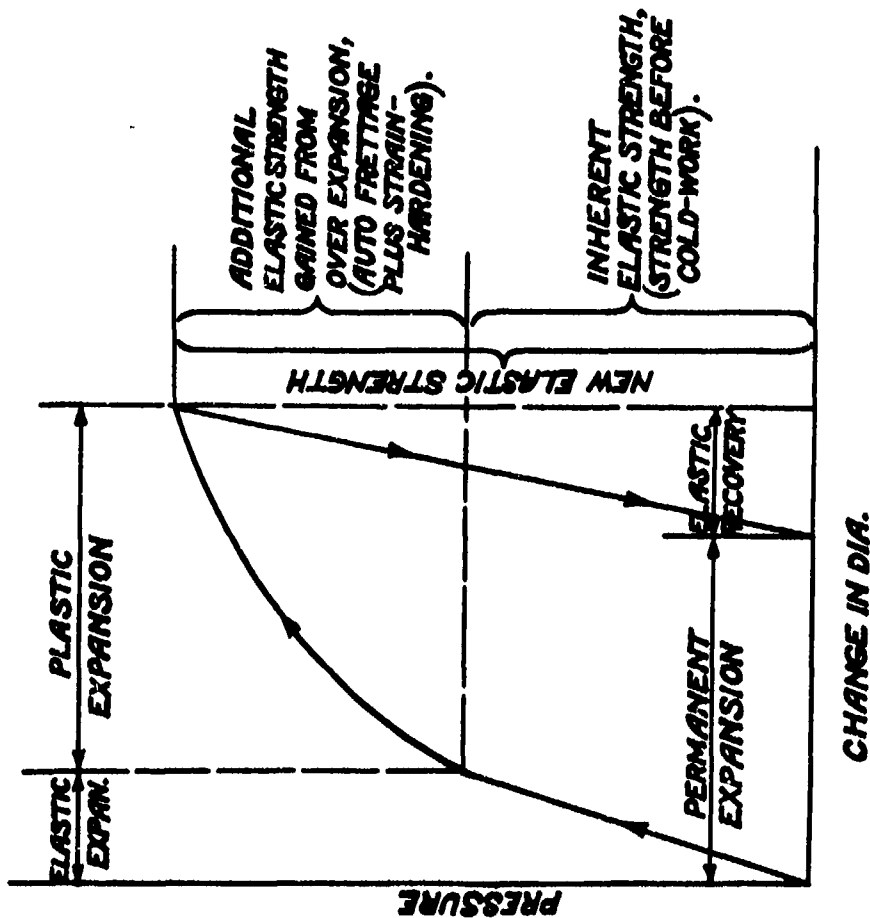
As an introduction to the behavior of cylinders in the plastic state, it may be of value to illustrate the effect of plastically expanding them with internal pressure. Fig. 12 (a) shows the general relationship between pressure and expansion. It is noted that additional elastic strength results from the overexpansion. Part of this gain is due to strain hardening and shows up as an actual increase in the yield strength which can be measured by tensile tests. The remainder of the gain results from the residual tangential stresses left in the material after the pressure is released. This is known as the autofrettage effect and is illustrated in Fig. 13. The term autofrettage comes from the French, a literal translation of which is "self hooping". From the figure, it is seen that there is a residual "hoop" of tension on the outside and a residual "hoop" of compression at the bore, thus producing the same effect in a monobloc tube as that which is obtained by either wire wrapping or shrinking tubes on liners.

The straight line in Fig. 12 (a) illustrating the recovery during the release of pressure is actually curved as shown in exaggeration by the solid line in Fig. 12 (b). A reapplication of pressure causes expansion along the reloading curve, shown also as a solid line, forming what is known as an hysteresis loop. When the material is subjected to a low temperature soak (300° C) before reapplying pressure, the loop closes to essentially a straight line as indicated by the dotted line.

The autofrettage process as used by the U. S. Navy and foreign nations involves lower pressures with smaller expansion than that employed by the U. S. Army. In those processes, strength is gained largely from autofrettage with little being gained from strain hardening. To differentiate, the Army generally speaks of its process as "cold working".

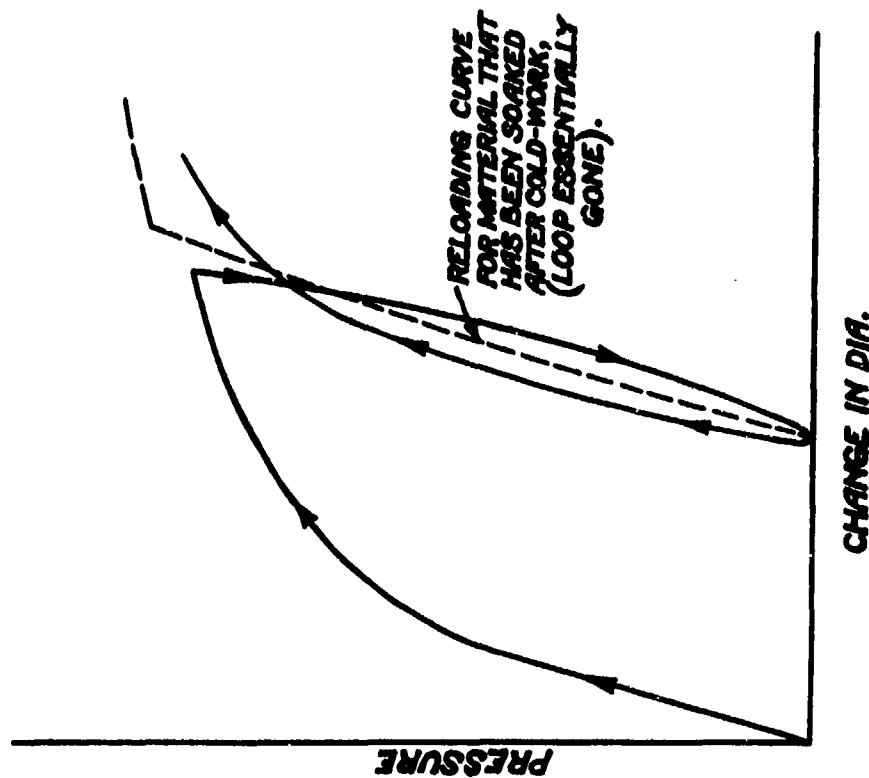
In the early 1920's, it was the desire of the Ordnance Department to thoroughly investigate the possibility of strengthening gun tubes by overstraining or plastically deforming them with hydraulic pressure. Development work was undertaken and from this, grew the cold working process, as now used at Watertown and Watervliet Arsenal. Hundreds of observations were made on overstrained thick hollow cylinders and data was correlated in such a manner that it was directly applicable to the design of cold worked gun tubes and high-pressure vessels.

The most important variables to be correlated in the development work were pressure, yield strength, wall ratio and deformation. Relationships were clearly expressed by pressure factor, flow factor and strain ratio, which are briefly explained as follows:



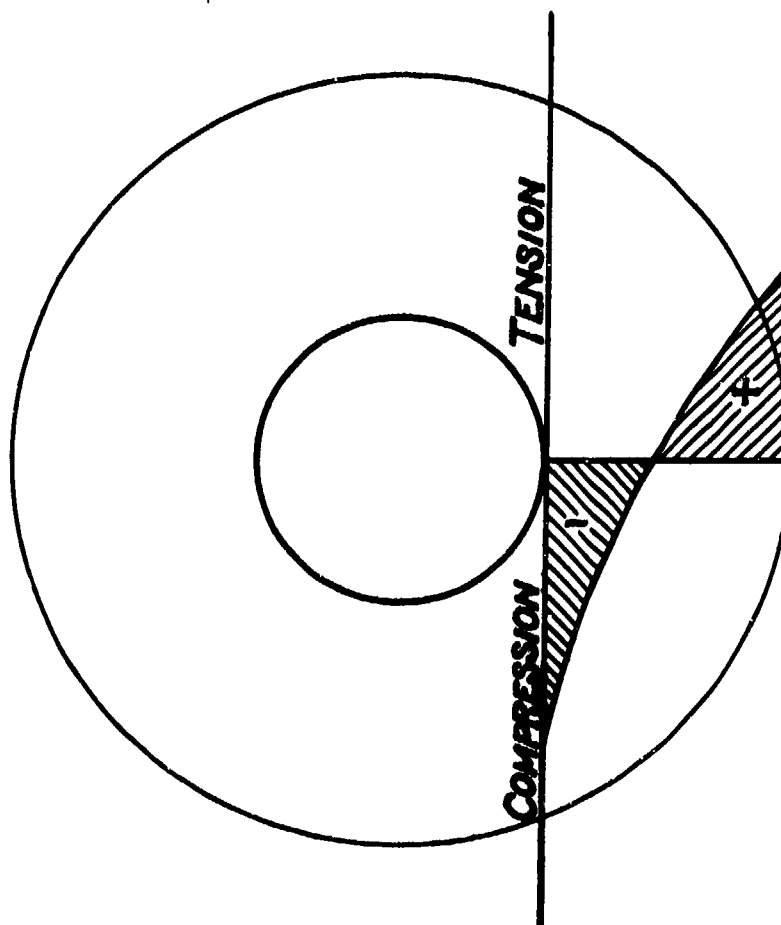
**PRESSURE-DEFORMATION CURVE
INDICATING GAIN IN STRENGTH
RESULTING FROM COLD-WORK.**

(a)



**PRESSURE-DEFORMATION CURVE
SHOWING EFFECT OF LOW-TEMP.
SOAK ON HYSTERESIS LOOP.**

(b)



RESIDUAL HOOP STRESSES
AFTER AUTOFRETTAGE

WTN.639-5857

FIGURE 13

- 1) Pressure Factor is a convenient dimensionless ratio, previously noted as P/S_{yp} , serving as a "Common Denominator" for comparing cylinders of various wall ratios and yield strengths with regard to the pressures required to produce any given degree of permanent enlargement.
- 2) Flow Factor is a ratio of the permanent enlargement of the inside of a cylinder to that of the outside.
- 3) Strain Ratio is the ratio of permanent strain on the inside of a cylinder to that on the outside. It is related to flow factor; however, it is expressed independently as it is frequently convenient to have it so.

Fig. 14 shows several interesting relationships concerning pressure factor. The top family of curves, based on original test data, concern various percentages of cold work. The six percent curve is emphasized by a heavy line as this percentage is used for production design. Immediately below the one percent curve, there is shown a line applicable to a hypothetical material that would not resist deformation after yielding; i.e., one that would not strain harden. The equation for the line, coming from the theory of plasticity, is $PF = L_n W$. In this, it is assumed that yielding has progressed throughout the wall of the cylinder and since the material does not strain harden it applies to autofrettage divorced from strain hardening. Still further below is a line applicable to yielding as determined by the maximum shearing stress theory. With pressure factor being directly proportional to pressure, these curves show relative strengths that may be attributed to the inherent elastic strength, autofrettage, and strain hardening, as indicated by the brackets shown.

In the following paragraphs, various equations concerning strain in the plastic state are given. Their derivations (not shown here) involve the following assumptions, listed as a point of interest:

- 1) Poisson's Ratio = $1/3$. For the relatively small values of strain encountered in autofrettage and cold working, experimental data and production experience shows this to be reasonable.
- 2) A condition of constant volume exists. This assumption is made since elastic strains are negligible compared with the plastic ones. As a result of this assumption, the sum of the principal strains is equal to zero.
- 3) The directions of the principal stresses and strains coincide.

WTN.639-5847

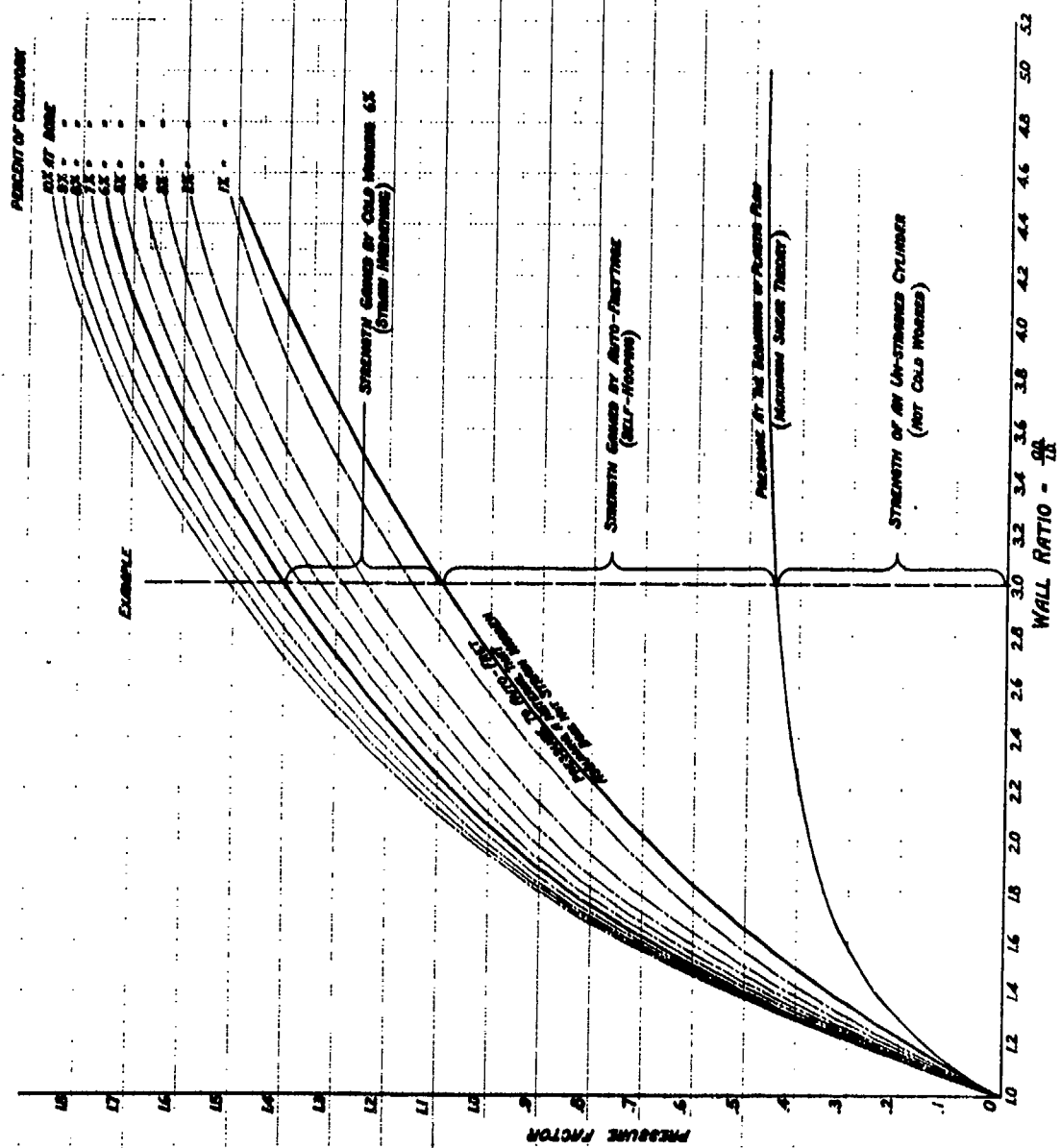


FIGURE 14

WATERTOWN ARSENAL SEPTEMBER 11, 1942
 DRAWN BY K. E. HARRIS

Fig. 15 shows plots of strain ratio and flow factor vs. wall ratio. The equation of the strain ratio curve is:

$$\frac{e_1}{e_0} = \frac{5}{6} W^2 + \frac{1}{6} \text{ --- (53)}$$

This is a slight modification of an empirical equation based on original experimental data. The value of having an expression showing the relationship between the inner and outer strains is obvious and it is fortunate that the relation is so simple. It is occasionally more convenient to have this relationship expressed in terms of flow factor.

$$FF = \frac{\Delta ID}{\Delta OD} = \frac{e_1}{e_0} \times \frac{ID}{OD}$$

and in terms of wall ratio this becomes

$$FF = \frac{e_1}{e_0 W} = \frac{5}{6} W + \frac{1}{6W} \text{ --- (54)}$$

This is the equation of the flow factor curve plotted.

Frequently it is desirable to calculate longitudinal strain. By taking accurate measurements during cold working of gun tubes and test cylinders, it has been found that longitudinal shrinkage is uniform across the section and from this observation, it appears reasonably sound to assume that longitudinal strain is similarly uniform. During cold working, gun tubes are free to contract longitudinally until the gun bears against the container. Until that instant, no radial stress exists on the outside surface and since no shrinkage occurs after that instant, radial stress subsequently developed does not effect the calculation of shrinkage. Only tangential stress, a uni-axial one, exists on the outside surface. It, therefore, is correct to express longitudinal strain as Poisson's ratio times the strain on the OD. By expressing e_0 in terms of e_1 and wall ratio (from Equation 53) and multiplying by μ the following expression for e_z is obtained.

$$e_z = \frac{-e_1}{2.5W^2 + .5} \text{ --- (55)}$$

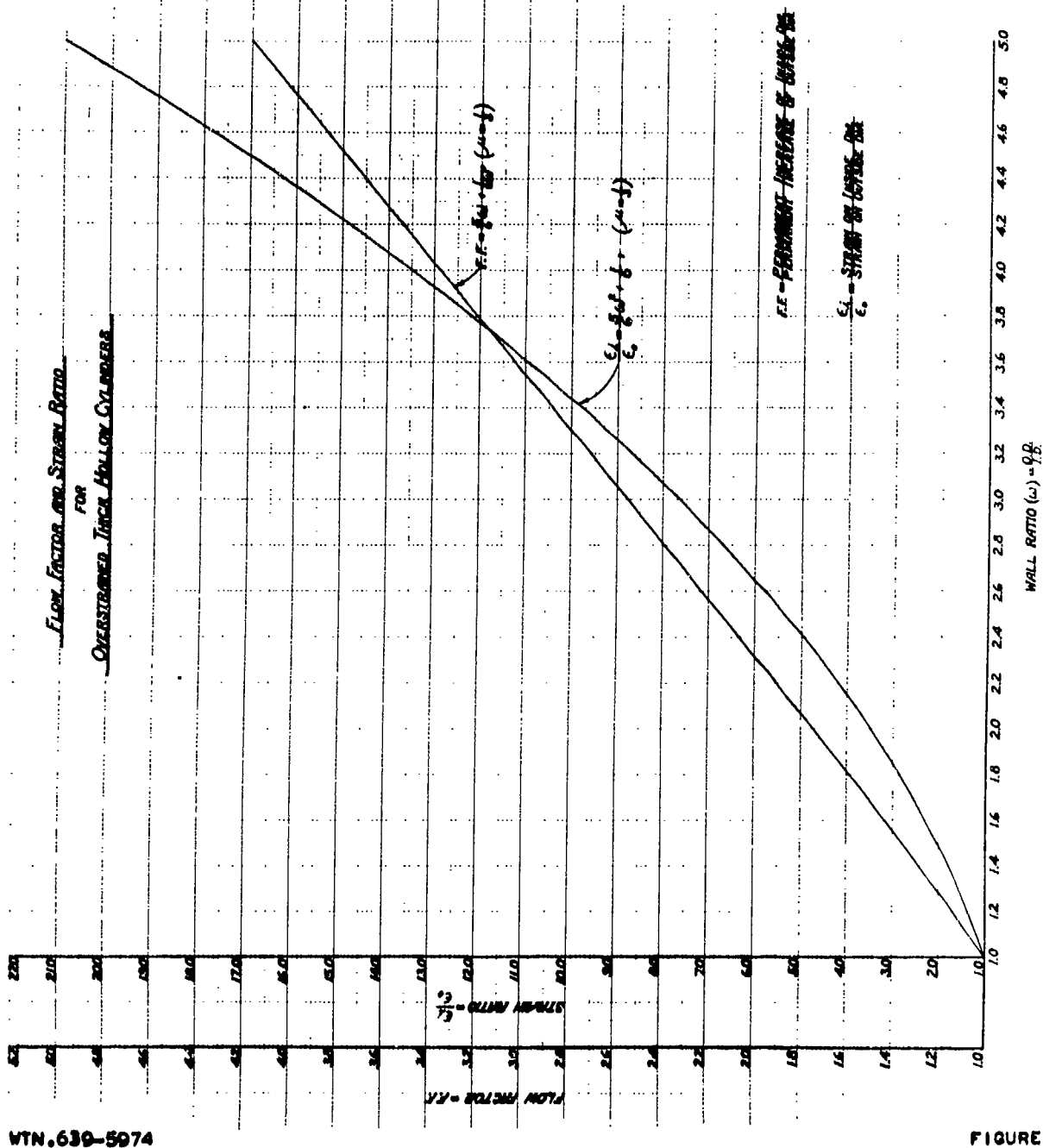
This equation is plotted in Fig. 16.

By having established the expression for longitudinal strain, shrinkage in cylindrical sections is simply the product of strain and length. The relationship, which has been derived, is considerably more complicated for a tapered section. This is shown in the following equation:

$$\Delta L = \frac{-.895 e_1 L}{W_b - W_m} \left[\tan^{-1} 2.236 W_b - \tan^{-1} 2.236 W_m \right] \text{ --- (56)}$$

Equations 54, 55 and 56 are based on the strain ratio equation number 53; which, as previously pointed out, is a modification of an empirical relation. The original empirical equation is:

$$\frac{e_1}{e_0} = \frac{W^2}{1.125} \text{ --- (57)}$$



WTN.639-5974

FIGURE 15

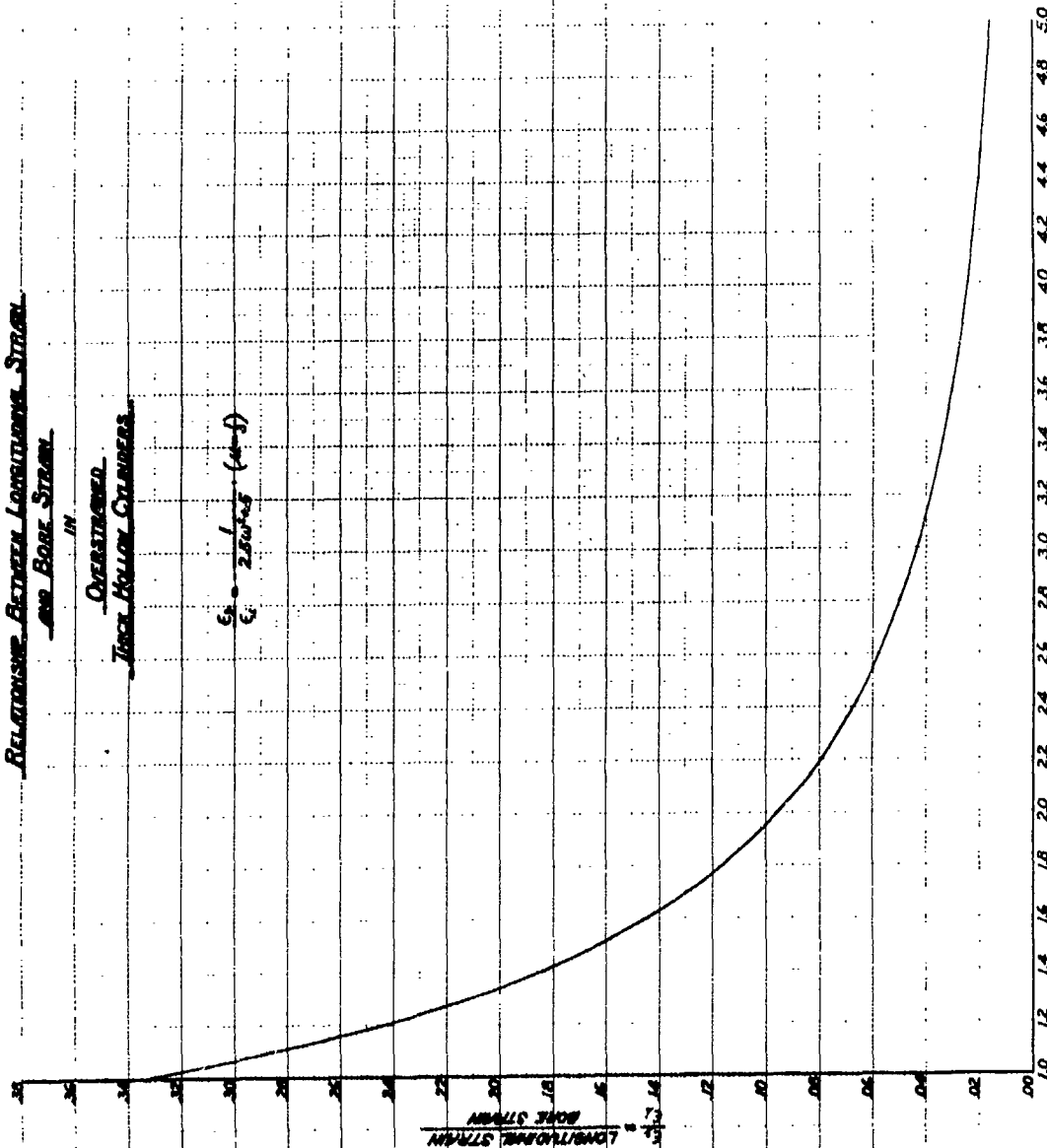
WATERTOWN ARSENAL NOVEMBER 26, 1943
Drawn by: J. Rowan
Checked by: J. R. P. H.

RELATIONSHIP BETWEEN LONGITUDINAL STRAIN
AND BORE STRAIN

IN

Observations
Thin Hollow Cylinders

$$\epsilon_s = \frac{1}{2} \epsilon_b (1 + \frac{r}{R})$$



MALL RATIO (ω) = $\frac{9}{16}$

WATERTOWN ARSENAL NOVEMBER 26, 1943
Drawn by J. J. Jones
Checked by J. J. Jones

WATER 63A, ENO

FIGURE 16

While equation 53 is more accurate than equation 57, the latter lends itself to less complicated expressions for flow factor, longitudinal strain and longitudinal shrinkage in tapered sections, as tabulated below:

$$FF = \frac{W}{1.125} \text{ (APPROXIMATE) } \text{-----} (58)$$

$$e_z = \frac{-0.375e_l}{W^2} \text{ (APPROXIMATE) } \text{-----} (59)$$

$$\Delta L = \frac{-0.375e_l \times L}{W_b \times W_m} \text{-----} (60)$$

Equations 57 to 60 inclusive are accurate to within 5% for all wall ratios except those below 1.3 and for such ratios these equations should not be used.

Formulae expressing (1) e_l in terms of e_o and (2) e_z in terms of e_l have been discussed above. From these, equations for tangential and radial strains have been derived.

TANGENTIAL STRAIN

$$\text{General } e_t = \frac{a^2 e_l}{r^2} - \left(1 - \frac{a^2}{r^2}\right) \frac{e_z}{2} \text{-----} (61)$$

$$r = a \quad e_t = e_l \text{-----} (62)$$

$$r = b \quad e_t = \frac{1}{W^2} \left[e_l - (W^2 - 1) \frac{e_z}{2} \right] \text{-----} (63)$$

RADIAL STRAIN

$$\text{General } e_r = -\frac{a^2 e_l}{r^2} - \frac{e_z}{2} \left(1 + \frac{a^2}{r^2}\right) \text{-----} (64)$$

$$r = a \quad e_r = (e_l + e_z) \text{-----} (65)$$

$$r = b \quad e_r = -\frac{1}{W^2} \left[e_l + (W^2 + 1) \frac{e_z}{2} \right] \text{-----} (66)$$

In proof testing, it may be desirable to overstrain the material throughout the wall. Since the outside surface would be the last to yield, the strain ratio may be applied to determine the required bore enlargement (percent of cold work) to proof test. The pressure required for proof may then be determined directly from Fig. 14 where pressure factors for various percentages of cold work are plotted. With it being frequently necessary to make such calculations, the strain ratio relationship has been conveniently plotted in Fig. 17. This family of curves is obtained by substituting S_{yp}/E for e_o in equation 53 and multiplying both sides of the equation by $100 S_{yp}/E$ in order to convert e_l to percent cold work.

WTN.639-5851

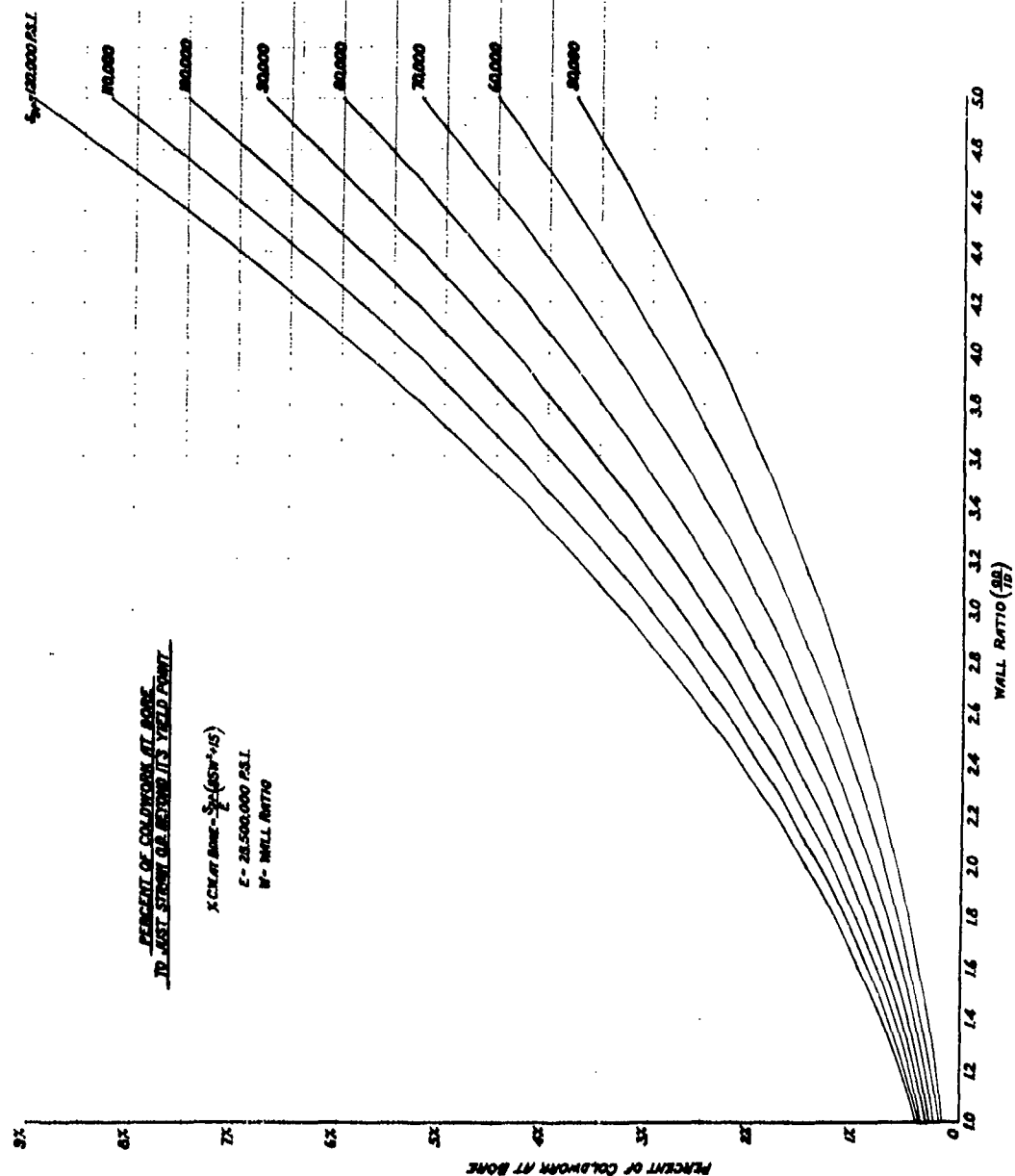


FIGURE 17

RUPTURE STATE

Knowledge concerning rupture can be of considerable value in design work. While literature discusses the subject to some extent, it is generally appreciated that much work remains to be done before the various factors contributing to rupture can be accurately evaluated. This is of particular importance in the design of gun tubes and high-pressure vessels in which factors of safety are unavoidably small. The following discussion neither reviews the literature nor contributes appreciably to this evaluation. It merely touches on some of the highlights in the field and presents some interesting observations made during high-pressure work at Watertown Arsenal. It is pointed out that rupture as considered here, may differ from that encountered in either the bursting of cannon under fire or in the fragmentation of bomb and shell because of the difference in rates of loading.

In general there are two broad classifications of rupture, brittle and ductile.

It has been observed that cylinders of inherently brittle material rupture in a radial plane as illustrated in Fig. 18, whereas those of ductile materials rupture along an equiangular spiral (the path of maximum shearing stress) as illustrated in Fig. 19. Fig. 20 is another photograph of the cylinder (shown in Fig. 19) prior to sectioning and is included to show the extent of that rupture.

Two interesting generalities concerning brittle materials have been observed. These are:

- 1) Cylinders of heavy wall ratio appear to rupture by separating across a diameter into two distinct halves with little if any fragmentation occurring. A little added ductility apparently causes rupture to occur as a radial crack running longitudinally along only one side of the cylinder and open only to the bore.
- 2) Rupture (not just yielding) occurs essentially in accordance with the maximum stress theory of strength. If stress raisers are present and not allowed for in computations, rupture may occur at even lower pressures than those computed. Test data for cast iron cylinders (by Cook and Robertson) expressed in terms of pressure factor is plotted against wall ratio in Fig. 21. The agreement with the maximum stress theory is obvious.

In contrast to the above, ductile materials may yield tremendously before rupture. Bridgman reports 300% bore expansion in a lead cylinder. Enlargements before rupture in steel cylinders of

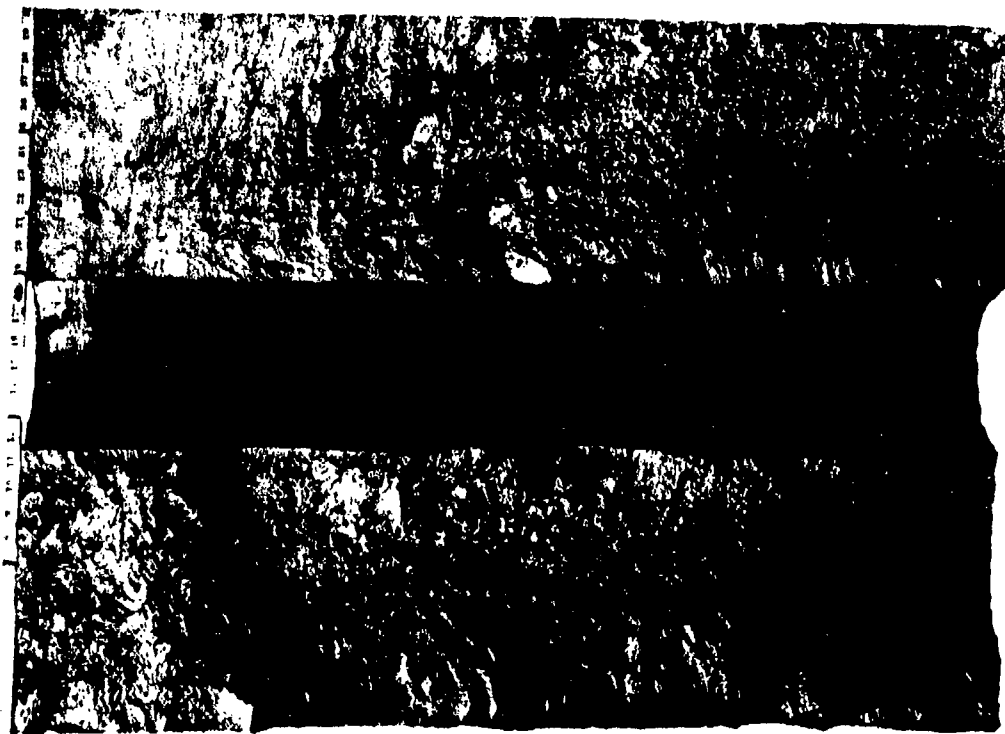


FIGURE 16

WATERTOWN ARSENAL

CAST STEEL COLD WORK CONTAINER SERIAL NO. 2
MFG. BY OHIO STEEL FOUNDRIES. FAILED UNDER
HYDROSTATIC TEST AUGUST 15 1942.
AUGUST 21 1942 W.A.660-113



INCHES 1 2 3

ORDNANCE DEPT. U.S.A.
WATERTOWN ARSENAL

REPRODUCED AT GOVERNMENT EXPENSE

FIGURE 19



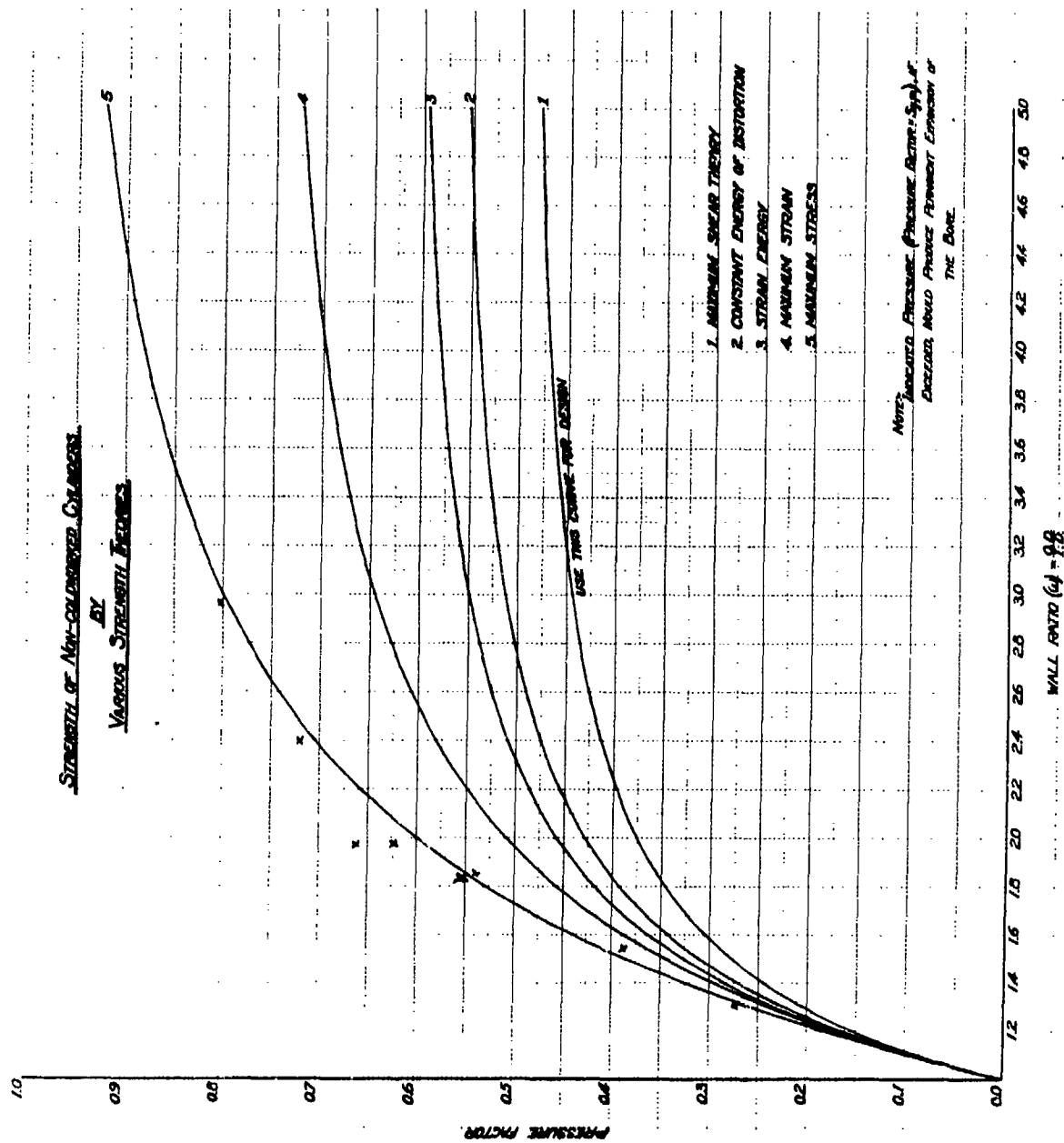
W A T E R T O W N A R S E N A L
C O N F I D E N T I A L

GENERAL VIEW, FRONT, OF TEST CYLINDER IN COLD WORK PRESS
AFTER RUPTURE. MAY 21 1942 W.A.660-85

FIGURE 20

WTN.639-5976

(x) POINTS OBSERVED BY COOK & ROBERTSON IN THE RUPTURE OF CAST IRON CYLINDERS BY INTERNAL PRESSURE.



from 40 to 50% are not uncommon. Unfortunately there appears to be no law of rupture comparable with that concerning brittle materials. It is pointed out however, that the pressure required for 10% cold working can be taken as a fair approximation of the pressure required to rupture ductile steels. The validity of this rule of thumb may be appreciated when one considers how flat the pressure expansion curve is at 10% cold working. A ductile cylinder first bulges where it is weakest and then, in the area of the bulge, ruptures along a spiral path.

The discussion above concerns rupture in material essentially homogeneous. Such factors as localized imperfections, (inclusions, hard or soft spots and the like) cracks, geometric discontinuities (re-entrant corners, bolt holes, etc.) and fatigue often cause an otherwise ductile cylinder to rupture in a brittle manner. Variation in structure, a metallurgical imperfection which may be seen in the photograph, accounts for the brittle failure illustrated in Fig. 22. Based on physical tests, this material was essentially ductile. (Reduction of Area = 40.2%) A crack on the outside surface accounts for the brittle rupture of the gun tube (Reduction of Area = 65%) illustrated in Fig. 23. From the photoelastic picture, Fig. 24, provided by the Arsenal Laboratory, one can appreciate the tremendous localized stress in the re-entrant corners of gun tube rifling. Fig. 25 is an illustration of brittle rupture resulting from fatigue. The piece, a liner in a high-pressure intensifier cylinder, experienced approximately 3000 stress reversals before rupture.

It is interesting to note that in a fracture, its origin is often clearly indicated by the pattern of the tear. Usually well defined "Feather lines" point toward the origin as shown in Fig. 26.



INCHES 1 2 3
ORDNANCE DEPT U.S.A.
WATERLOO ARSENAL
CYLINDER 1-1-1943 WATER BOMB
PULPING GILD WORKING.
November 4, 1943

FIGURE 22

WATERTOWN ARSENAL
105 mm HOWITZER C6516 ruptured
during Cold Working.
February 21, 1941 W.A.660-62

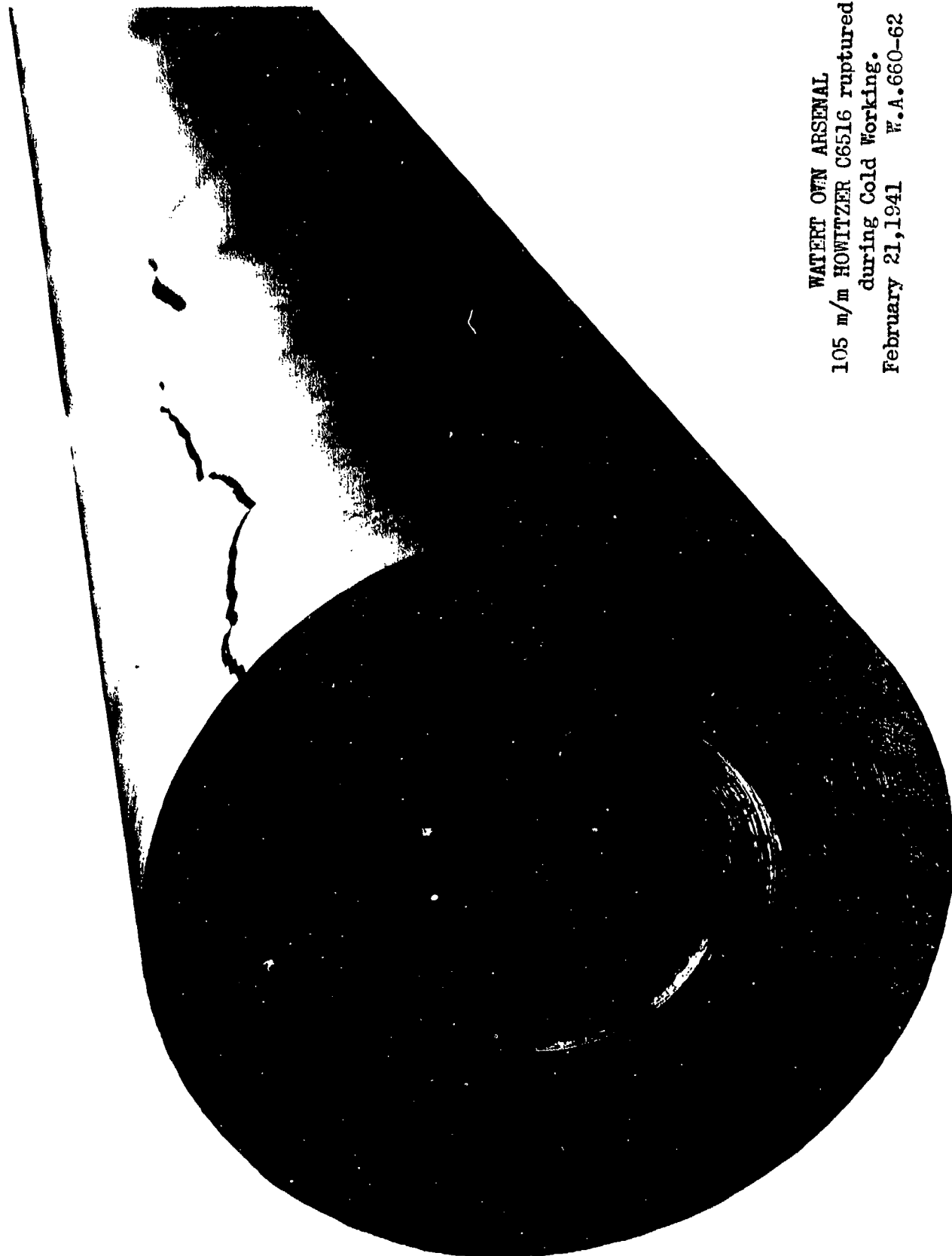
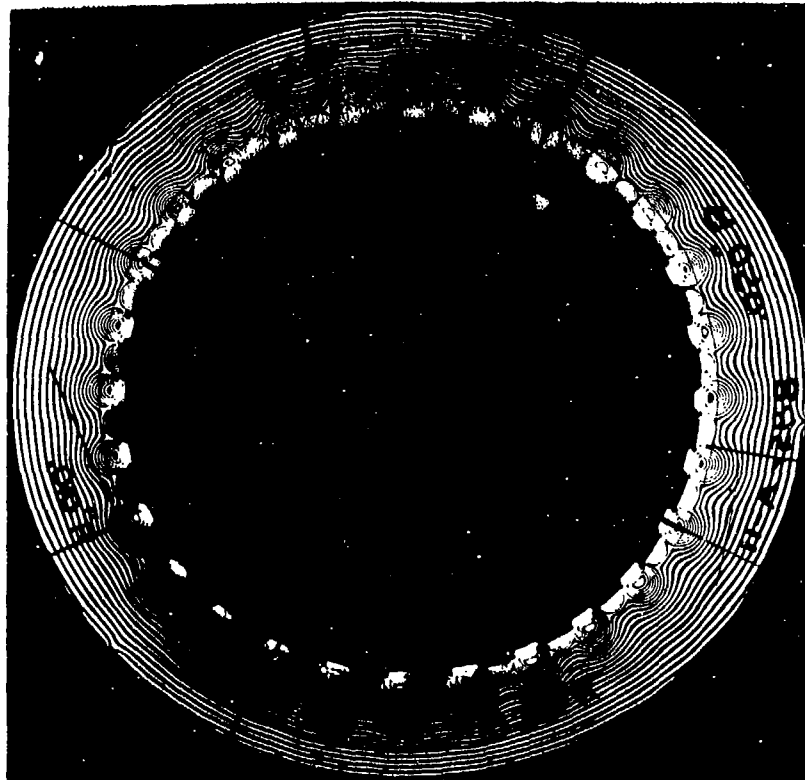
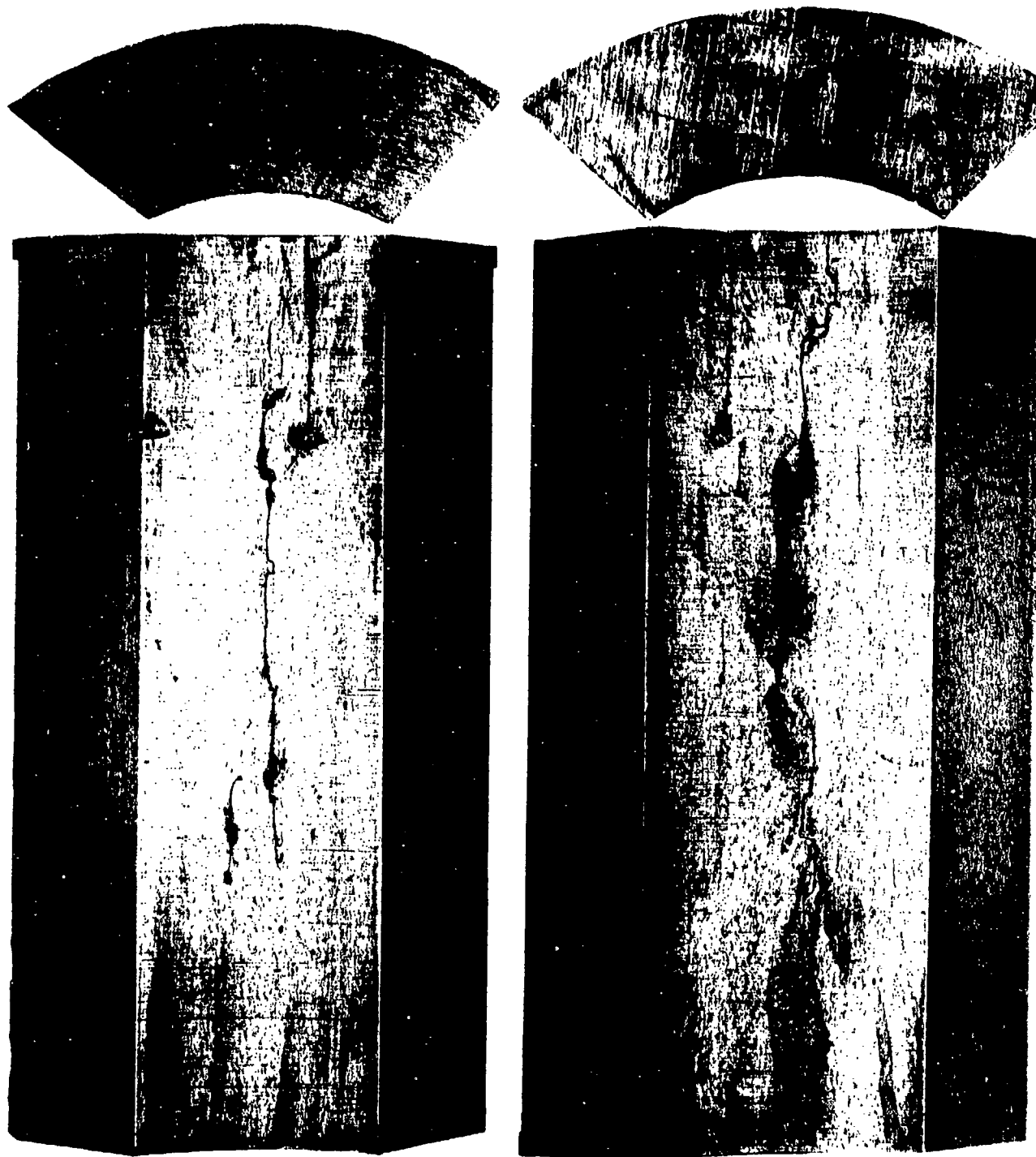


FIGURE 23

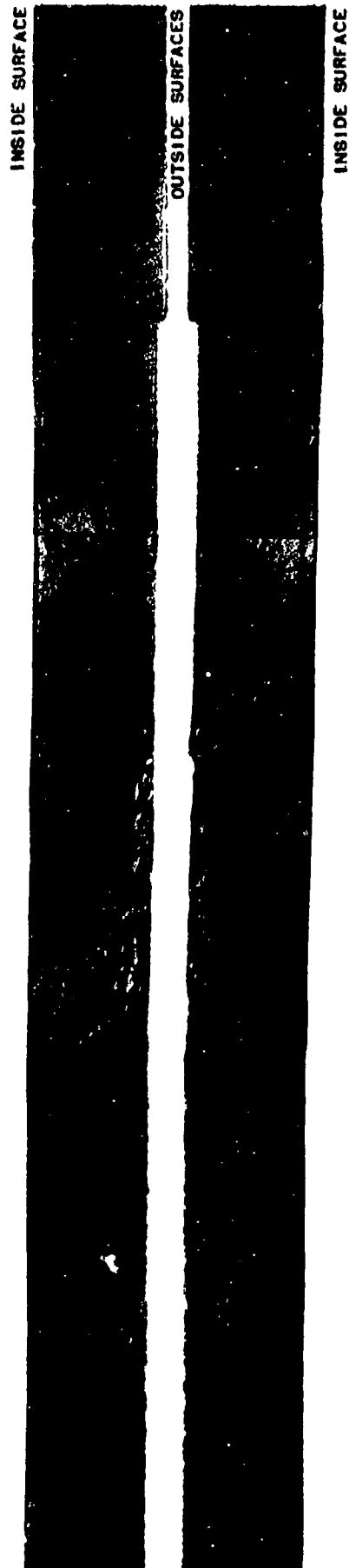


Photoelastic Picture Showing Localized Stress
In The Re-entrant Corners of Gun Tube Rifling
WTN.639-6037

FIGURE 24



FATIGUE CRACKS IN A HIGH PRESSURE CYLINDER FROM REPEATED LOADS
NOVEMBER 1943
WTN.639-5977



WATERTOWN ARSENAL
FRACTURE SURFACES INCLUDING THE ORIGIN OF FAILURE IN A GUN TUBE WHICH PROVED
DEFECTIVE DURING COLD WORK AT WATERVLIET ARSENAL. ORIGINAL PHOTO MADE AT
WATERTOWN ARSENAL. VTN.639-5972